# Mathematical Access Worries and Accounting for Knowledge of Logical Coherence

Sharon Berry

http://seberry.org

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# Outline

# I. Introduction

# The Access Problem

**Mathematical access problem** (roughly): If there are objective proof-transcendent mathematical facts, how could human knowledge of mathematics be anything but a miracle or a mystery?

- Sometimes this worry is motivated by noting that we have no causal contact with mathematical objects.
- But the intuitive problem isn't limited to the Platonist.

# Agenda

In this talk, I will sketch an answer to this mathematical access problem. But first we must zoom in a little bit on

- how to understand access worries
- what it takes to solve them

# Mathematical Access Problem in More Detail

Access worries appeal to informal coincidence avoidance/reduction intuitions that are widely accepted and fruitful in the sciences.

A realist theory of some domain **faces an access problem** to the extent that adopting it[1] seemingly forces one to posit a match between human psychology and certain belief-independent facts about that domain, which

- intuitively cries out for explanation
- but goes unexplained.

[1] rather than comparably attractive less realist alternative views on the domain in question

# A 'How Possibly' Question

So the access problem poses a kind of 'how possibly' question:

- how could the realist possibly explain human accuracy about mathematics (without appeal to some significant 'extra' coincidence)?

# Answer By Toy Model

Accordingly, one can solve the mathematical access problem by providing a **toy model**[2]: a sample explanation for human accuracy about mathematics which

- includes all the features of our situation that seem to make adequate explanation impossible
- but may be simplified in other ways
- removes the appearance that accepting mathematical knowledge commits us to positing some significant extra coincidence

[2] See the literature on 'how possibly' questions [2, 9].

And that's what I'll try to do...

# Step 1: Reducing the Access Problem

# Step 1: Reducing the problem

To begin I'll

- note a certain aspect of mathematical practice
- which inspires a structuralist consensus that mathematical access worries can be reduced to access worries about logical possibility.

# Mathematicians' Freedom Intuitions

Mathematical practice seems to allow use of any logically coherent pure mathematical posits.

> *Reflecting on my experiences as a research mathematician, [some] things stand out. First, the frequency and intellectual ease with which I endorsed existential pure mathematical statements and referred to mathematical entities. Second, the freedom I felt I had to introduce a new mathematical theory whose variables ranged over any mathematical entities I wished, provided it served a legitimate mathematical purpose. [3]*

# Structuralist Consensus

**Structuralist Consensus:** Mathematicians can introduce (almost) any logically coherent stipulations defining a pure mathematical structure they wish because...

- **Modal Structuralist:** mathematical claims really express modal conditionals "it would be logically possible for there to be objects satisfying such- and-such axioms, and necessarily, if there were objects satisfying such-and-such axioms then ..." $\Diamond D \wedge \Box(D \rightarrow \phi)$
- **Plenitudinous Platonist:** the mathematical universe is very large, e.g. all structures of interest can be identified with certain sets (c.f. Bourbaki)
- **Quantifier Variantist:** we have some freedom to choose how our language will 'carve up' the world into objects.

# All Done?

Does this solve the mathematical access problem?

- Not quite! The structuralist consensus only says that all *logically coherent* structures may be introduced.
  - If mathematicians adopted subtly syntactically inconsistent stipulations like Frege's set theory with Basic Law 5, they wouldn't speak the truth
- So mathematicians' ability to recognize logically coherent axioms can still seem to raise an access problem/still needs to be explained.

Note: pure first order logical deduction can't establish even very basic claims about logical coherence like

- $\Diamond(\exists x)(\exists y)(\neg x = y)$

# Note On Logical Possibility

When evaluating logical possibility we:

- consider only the most general constraints on how any relations could possibly apply to any objects (c.f. Frege)
- ignore all limits on the size of the universe
- ignore all subject matter specific constraints
  - e.g., $\Diamond \exists x(\mathit{Raven}(x) \wedge \mathit{Vegetable}(x))$ is true.

.

Logical possibility is interdefinable with entailment

- e.g., $\Diamond\phi$ iff $\neg\phi$ isn't entailed by the empty premises.

# Why accept $\Diamond$ as a Logical Primitive? I

Why accept $\Diamond$ as primitive logical vocabulary (rather than analyzing it in terms of the existence of set models)?

- (Boolos, Field) Intuitively, if $\phi$ is logically necessary it must be true in the actual world. [6, 8, 1, 4, 5]
  - But the mere absence of set counter models (which might, e.g., have certain limits of size not relevant to the whole of reality) doesn't generally or clearly ensure this.
  - though the completeness theorem happens to ensure this when $\phi$ is in FOL.
- (Boolos) "one really should not lose the sense that it is somewhat peculiar that if G is a logical truth, then the statement that G is a logical truth does not count as a logical truth, but only as a set-theoretical truth"[1].

# Why accept $\Diamond$ as a Logical Primitive? II

- We shouldn't take set theory to be more fundamental just because of the historical accident of it currently being more developed. If anything
  - Conceptions of the hierarchy of sets are often explained using modal notions 'Successor stages contain sets corresponding to 'all possible ways of choosing' sets from stages below'
    - (we'll see such claims are easily formulable with the conditional logical possibility operator $\Diamond_{...}$ below)
  - Height arbitrariness worries may independently motivate potentialist set theory which analyze set existence in terms of logical possibility, rather than the other way around

# Step 2: Knowledge of Logical Possibility

# Agenda

In the following sections I'll try to

- solve the remaining access problem of knowledge of logical possibility
- by providing a toy model which explains how creatures like us (in all ways that generate intuitive access worries) could have gotten suitably powerful reliable methods of reasoning about logical possibility[3].

[3] i.e. dispositions to make **largely correct rather than incorrect** judgments about logical possibility and impossibility, insofar as we are inclined to make any judgments at all.

Specifically, I'll

- Sketch how could explain a degree of human accuracy about the logical coherence of **first order logical** states of affairs.
- Address some possible objections to this basic story.
- Extend this basic story to account for knowledge of $\Diamond\phi$ claims where $\phi$ includes non FOL vocabulary powerful enough to categorically describe structures like the natural numbers

# Initial Non-Mathematical Faculties

Imagine creatures who speak a language much like our own and already have the following widely accepted non-mathematical faculties:

- first order logical deduction,
- broadly accurate sensory perception of non-mathematical objects
- and general methods for good scientific reasoning (including abduction/inference to the best explanation).

# Acquiring Logical Possibility Knowledge

How could these creatures acquire knowledge of logical possibility claims $\Diamond\phi$?

Remember: pure first order logical deduction can't establish even very basic claims about logical coherence like

- $\Diamond(\exists x)(\exists y)(\neg x = y)$

# Initial Minimal Logical Possibility Knowledge

I take that it wouldn't be massively surprising (in the sense relevant to access worries) if such creatures acquired a minimal notion of logical possibility assumed to satisfy the two schemas below

- What's actual is logically possible ($\phi \rightarrow \Diamond\phi$)
- Logical possibility treats all n-place relations the same ($\Diamond\phi \leftrightarrow \Diamond\phi[S_1/S'_1 ... S_m/S'_m]$)[4].
    - e.g., If it's logically possible that a dog licks itself if and only if it's logically possible that a camel bites itself.

[4] When $S_1 \ldots S_m$ and $S'_1 \ldots S'_m$ are all distinct relations with each $S'_i$ having the same arity as $S_i$ and no $S'_i$ occurs in $\phi$.

# Big Picture

I'll suggest that

- We can get initial data points about logical possibility from the fact that what's actual is logically possible.
- Scientific-induction-like generalization[5] could then lead to good methods of reasoning about logical possibility.

Thus, our knowledge of logical possibility is ultimately no more mysterious than our our knowledge of physical or chemical possibility.

[5] whether at the level of evolution, cultural selection or individual experience

# I. Inference from $\phi$ to $\Diamond\phi$

Inference from actual to logically possible gives us some initial data points regarding what's logically possible.

- if $\phi$ then $\Diamond\phi$ (and the same goes for all substitution instances)

# I. Inference from $\phi$ to $\Diamond\phi$

For example, suppose that you aren't sure whether some mathematical hypothesis involving relations $P$, $Q$, and $R$ is logically possible.

- If you then note that the relations of friendship, nephew-hood and having been in military service together apply in just this way to the royal family of Sweden, this will get you to accept that the scenario in question is, indeed, logically possible.

# II. Generalization to $\neg\Diamond$ facts

Noticing patterns in non-mathematical reality can also teach us that certain things are logically impossible.

Suppose, for example, that someone thought it was logically possible for 9 items to differ from one another in which of three properties they have, e.g., for 9 people to choose different combinations of sundae toppings from a sundae bar containing three toppings.

- That person would have to somehow explain the striking law-like regularity that, regardless of the type of items and properties, we never wind up observing more than 8 such objects.

# IBE to Logical (vs. Physical or Metaphysical) Possibility

Note: One could instead explain this regularity by positing new physical or psychological laws to explain why free choice of sundae toppings never generated the forbidden 9th possible outcome.

But these laws would *also* have to explain why the analogous regularity held

- at every physical scale we can observe, from relationships between the tiniest particles to relationships between planets and stars
- with regard to much less concrete subject matter like poems or countries[6]

[6] e.g. Try as you may, you will never manage to think up a poem with 9 different stanzas, each of which differs from the all the others in regard to which of three poetic themes it mentions.

# II. Generalization to $\neg\Diamond$ facts

Bigger picture:

We attempt to explain patterns in what actually happens by appeal to some combination of:

- general constraints on what's logically possible/necessary for any objects and relations
- subject matter specific metaphysical, physical laws about the properties and relations in question.

In some cases, considerations of theoretical elegance will favor explanation by appeal to logically necessity.

# III. Reflection and Generalization

Just as when reasoning about physical possibility/laws, reflection and scientific generalization from these initial data points could lead one to

- accept new laws and expand methods of reasoning by generalization/IBE from these datapoints.
- drop principles when they are found to conflict with data points and more firmly entrenched laws (e.g., implying that something known to be actual isn't logically possible).

# III. Reflection and Generalization

Note: the kind of elegant generalization which we see in the sciences (and which I want to invoke) goes beyond simple inferences like: ‘the sun rose every day for the past billion years, so it will rise tomorrow.’

- It can include the kind of, seemingly astonishing, leaps we see in the sciences like going from observations of points of light in the night sky to a whole model of how the planets are arranged

# III. Reflection and Generalization

Gödel famously suggested that such scientific generalization could support a choice of additional axioms for set theory.

> *"There might exist axioms so abundant in their verifiable consequences, shedding so much light upon a whole field, and yielding such powerful methods for solving problems... that, no matter whether or not they are intrinsically necessary, they would have to be accepted at least in the same sense as any well-established physical theory"[7]*

# Putting The Whole Toy Model Together

So, putting this all together we have a story about how creatures like us (could have) gone from

- knowledge of initial data points involving the logical possibility or impossibility of first order logical states of affairs (via inference from $\phi$ to $\Diamond\phi$ or generalization/IBE to $\neg\Diamond\phi$ as above)
- to use of accurate general methods of reasoning about logical possibility
- to recognition of logically coherent pure mathematical axioms
- to reliably true mathematical beliefs.

# Objections and Replies

# Worry 1: Reliability of Scientific Induction

Some doubt that scientific induction/IBE is ever reliable when applied to mathematics

But note:

- Mathematicians frequently use hunches based on past experience and the results of computational searches to guide their research.
    - Belief in Fermat's last theorem was partially the result of a consistent failure to find a counterexample.
- If we take this practice seriously, we can't reject appeals to scientific induction/IBE in mathematics as totally unreliable.

# Worry 2: Gap Between Finite and Infinite

A second worry goes like this: Our initial data points all concern finite structures of non-mathematical objects. But there's a big gap between the finite and the infinite.

- Many elegant generalizations that hold for finite collections fail for infinite structures, e.g., consider Hilbert's hotel.
  - You can make room for a new guest at Hilbert's hotel by moving each guest over one.
  - You couldn't do this for any finite sized hotel.

So scientific induction/IBE can't take us from knowledge of finite structures to knowledge of what's logically possible for infinite structures.

# Worry 2: Gap Between Finite and Infinite

Response 1 (Contentious):

- Some widely accepted claims about non-mathematical objects do require the existence of infinitely many things.
    - e.g. 'For every segment of space in the path of Zeno's arrow, there's a strictly shorter segment of space...[7]'
- So perhaps we know that (at least countably infinite) spatial paths exist
    - in the same way that we know about what holes, shadows, poems and commercial entities exist
    - regardless of what physics decides about the fundamental ontology underlying our spatial talk.

[7] and 'being shorter than' is transitive and antireflexive

# Another Source of $\Diamond\phi$ Knowledge?

Response 2: We can also add a secondary (reliable, if not infallible) source of $\Diamond\phi$ knowledge to the basic story above.

Many philosophers think that of some axioms enjoying either

- long use without deriving a contradiction
- scientifically explanatory use

is a (ceteris paribus) reliable sign of their *logical coherence*

- if not (as Quine might have it) their truth.

Thus, we might invoke the scientific explanatory usefulness/long use of a theory $\phi$ as a reliable (not to say infallible) source of logical possibility beliefs.

# Worry 3: Fading Out

Third, one might worry that principles of reasoning we generalize from small collections don't provide *enough* knowledge of logical possibility to make sense of the mathematical knowledge we seem to have.

- Our dealings with objects in the world tend to involve finite (or relatively small infinite) collections, e.g., gingerbread cookies, spatial paths.
- However, we claim to have knowledge about (the logical possibility of) large infinite collections, e.g., a hierarchy of sets satisfying $ZFC_2$.

# Large Sets & Canadian Fowl

A critic might advance the following analogy:

> *Saying that applying abduction and IBE to data points about what's logically possible for small collections yields laws correctly describing what's logically possible for larger collections is like saying that abduction and IBE from knowledge gained by observations of birds in California allows us to learn about birds in New York as well.*

# Limited Knowledge

I accept this analogy, and claim that it actually fits the current state of human knowledge with regard to facts about the higher infinite rather well.

- We can know *some* things about birds in New York just by inference to the best explanation from the facts about the birds in California.
- Our expectations about birds in distant locales are relatively **sparser** and **less confident** relative to our beliefs about birds in locations that we have observed.

But, this is just what happens with regard to our knowledge of what's logically possible with regard to large collections:

As one goes from claims about finite collections to countable collections (like the natural numbers), to uncountable collections (like the reals), to even larger collections mathematicians' beliefs **do** appear to get

- sparser, e.g., the continuum hypothesis (CH) is independent of ZFC despite its focus on relatively small sets.
- less confident: mathematicians are more confident in their claims about numbers, sets of numbers and sets of sets of numbers than in the distinctive claims of higher set theory.

Thus, I think this third worry actually points to a benefit rather than a flaw of the account at hand:

- this toy model predicts (and thereby maybe helps explain) the way that our knowledge of the mathematical objects does appear to thin out.

# Knowledge of Logical Possibility of Non-FOL Conceptions

# Knowledge of Logical Possibility of Non-First Order Claims

**Objection**: OK maybe this can explain our accuracy about $\Diamond\phi$ claims, where $\phi$ is a first order sentence. But that's not enough to solve access worries.

But to account for the kind of mathematical knowledge typical truthvalue realists think we have, we'd need to account for

- knowledge of logical coherence of axioms powerful enough to categorically describe math structures like the natural numbers
  - e.g., knowledge of facts like $\Diamond PA_2$.

# Partial Idea & Problem

- I've suggested generalization from initial knowledge of non-mathematical FOL facts can yield general principles/reasoning methods that recognize the logical possibility of first order states of affairs.
- Idea: Maybe similar generalization can explain knowledge of logical possibility of second order states of affairs (the sufficiently strong language required above) given initial data points what second orders states of affairs are actual.
- But how do we know what second order states of affairs are actual?

# Can't Presume Accuracy About Second Order Facts

Knowledge of second order facts can seem mysterious in all the ways knowledge of mathematical objects does. After all

- It's not like we can just "see" second order facts about physical scenarios, like seeing sets of eggs floating over an egg carton.

- If we want to solve the access problem, we can't presume accuracy about concrete second order facts.

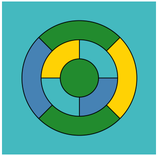
# Solution Step 1: Generalizing the Notion of $\Diamond$

Instead of appealing to second order logic, I will invoke a (similarly powerful) notion of

- **conditional/structure preserving logical possibility** $\Diamond_{R_1..R_n}$ (and corresponding notion of logical necessity $\Box_{R_1,..,R_n}$) whihc
  - generalizes the notion of logical possibility/necessity
  - makes claims about logically possible *given relevant structural facts about how relations* $R_1, .., , R_n$ *apply*.

To motivate this notion, consider what's natural to say in the following cases:

# A Motivating Example: Three Colorability

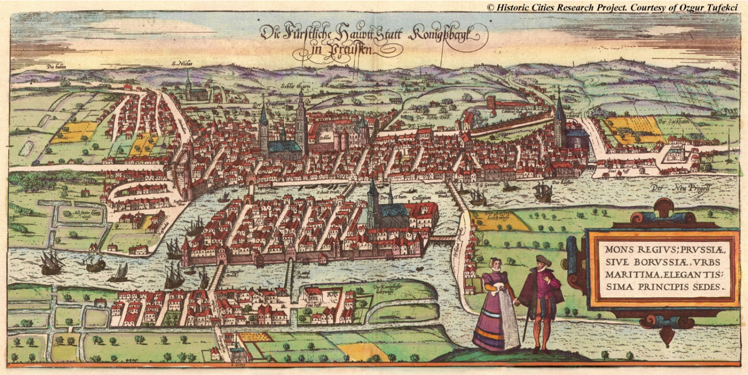


We might say:

- This map is not three color*able*.
- It's logically impossible, given the (structural) facts about how adjacency and and countryhood apply, that each country is either yellow, green or blue and no two adjacent countries are the same color.
- $\neg\Diamond_{\text{country,adjacent to}}$ [each country is either yellow, green or blue and no two adjacent countries are the same color.]

# A Motivating Example: Köeningsburg Bridges

We can also think about the famous property of the Königsberg bridges (that there's no way of traveling over each bridge exactly once) in these terms.



# How this Generalizes Logical Possibility

When evaluating $\Diamond_{R_1,\ldots,R_n}$ claims (as when evaluating $\Diamond$ claims) we:

- ignore all limits on the size of the universe
- consider only the most general combinatorial constraints on how any relations could apply to any objects (c.f. Frege).
- ignore all subject matter specific constraints on how different relations apply so that, e.g., $\Diamond\exists x(Raven(x) \wedge Vegetable(x))$ comes out true, even though it is metaphysically impossible for anything to be both a raven and a vegetable.

But we also hold fixed (structural facts about) how all the subscripted relations $R_1, \ldots, R_n$ apply

# Nesting Claims about Conditional Logical Possibility I

Consider claims like,

> **C&B**: *'It is logically impossible, given what cats and baskets there are, that each cat is sleeping in a basket and no two cats are sleeping in the same basket.'*

There's an intuitive sense of 'logically impossible' on which this claim will be true iff *there are more cats than baskets* in the actual world. We express this as follows:

- $\neg\Diamond_{cat,basket}$ each cat is sleeping in a basket and no two cats are sleeping on the same basket.'

# Nesting Conditional Logical Possibility II

We can also make claims about the logical possibility or impossibility of claims like **C&B** , saying things like

- $\Diamond$**(C&B)**: It would be logically possible for cathood and baskethood to apply in such a way that it would be logically impossible, given what cats and baskets there are, for each cat to sleep on a different basket.
- $\Diamond(\neg\Diamond_{cat,basket}$ each cat is sleeping in a basket and no two cats are sleeping on the same basket.')

This claim, $\Diamond$**(C&B)**, is true because:

# Nesting Conditional Logical Possibility III

$\Diamond(\neg\Diamond_{cat,basket}$ *each cat is sleeping in a basket and no two cats are sleeping on the same basket.')*

This claim, $\Diamond$(C&B), is true because:

- It's logically possible (holding fixed nothing) that there are 4 cats and 3 baskets.
- Relative to the scenario where there are 4 cats and 3 baskets, it's not logically possible, given what cats and baskets there are, that each cat slept on a basket and no two cats slept in the same basket.

We can use this notion to replace second order quantification in our categorical conceptions of mathematical structures:

For example we can express claims like second order induction:

$$(\forall X)\,[(X(0) \wedge (\forall n)\,(X(n) \rightarrow X(n+1))) \rightarrow (\forall n)(X(n))]$$

- **Induct**: 'It is logically necessary, given how number and successor apply, that if 0 is happy and the successor of every happy number is happy then every number is happy.'
    - $\Box_{\mathbb{N},\mathit{successor}}$[If 0 is happy and the successor of every happy number is happy then every number is happy]
- Note that **Induct** implies that if 0 is green, and the successor of every green number is green, then all numbers are green.

# $PA_\Diamond$

- We can write a sentence $PA_\Diamond$, (purely in terms of logical possibility) which categorically describes the natural numbers.
  - $PA_\Diamond$ resembles PA but replaces all instances of the induction schema from PA with the Induct$_\Diamond$ principle above.
- This ensures that for every sentence of number theory $\phi$, either $\phi$ or $\neg\phi$ is a logically necessary consequence of $PA_\Diamond$.
  - i.e., $\Box(PA_\Diamond \rightarrow \phi)$ or $\Box(PA_\Diamond \rightarrow \neg\phi)$
- Thus, it is enough to explain our knowledge of (facts like) $\Diamond\text{PA}_\Diamond$

# Solution Step 2

I propose that

- IBE on regularities involving first order truths about concrete objects could give some initial knowledge of conditional logical possibility ( $\Diamond_{R_1 \ldots R_n} \phi$) facts
  - e.g. the best explanation for the fact that no one ever actually takes a Kögnigsburg bridge walk (using each bridge exactly once etc.), is that it would be logically impossible to do so, given the facts about how bridges connect the landmasses.
- Given these initial data points, abduction/IBE as above can yield good general a priori methods of reasoning about nested conditional logical possibility claims – capable of recognizing facts like $\Diamond \mathrm{PA}_{\Diamond}$ and thereby answering mathematical access worries.

# Conclusion

# Conclusion

In this talk I've outlined a style of response to mathematical access worries which

- appeals to the Structuralist Consensus to reduce access worries about math to access worries about logical possibility.
- answers access worries about knowledge of logical possibility by providing a toy model for how such knowledge could have arisen.
    - suggesting knowledge logical possibility raises no more worries than knowledge of physical or chemical possibility.

# Contrast with Quine

(I think) this proposal avoids some problems for its closest relative in the literature

- Quinean empiricism about mathematics: We can get mathematical knowledge by accepting those mathematical objects that are indispensable to our best physical theories.

# Contrast with Quine

By taking experience and scientific generalization (or the like) to lead us **to good general methods of reasoning about logical possibility**, rather than directly telling us which math objects exist, it avoids Quine's problems allowing that:

- We seem to be able to learn about mathematical objects that have never been (and perhaps will never be) used by the sciences.
- No particular choice of mathematical fundamentalia seems strongly motivated by scientific applications (and physicists don't seem to care).
- Quantification over mathematical objects might turn out to be entirely dispensable (c.f. Hartry Field).

# No Commitments re: Empiricism or Innateness

This story also doesn't commit us to any position on the innateness of relevant mathematical/logical possibility reasoning.

- The kind of abductive generalization and correction by experience discussed above could (in principle) happen via
  - conscious adult IBE and belief revision
  - meme selection (c.f. countries dutch booking themselves when developing probability theory)
  - natural selection on fairly hardwired propensities to think, rather than millian experience

# Epistemic Stockholm Syndrome

The story I've proposed also doesn't commit us to empiricism about mathematical knowledge.

For note: even when experience and IBE play a clear role in getting us to adopt new methods of logical mathematical reasoning, we can (and do) sometimes still

- take deploying these new methods to yield a priori knowledge
- and say that we should have reasoned this way all along a priori

# Epistemic Stockholm Syndrome

For example, the online supplement to a New York Times article [10] gave readers

- experiences with a computer simulation (using a random number generator) of playing the Monty Hall game hundreds of times
- that changed their minds about which methods of reasoning about probability are appropriate for use *a priori*

# Appendix

# Mimicing Nested Conditional Logical Possibility

Using set theory, we can approximately mimic truth conditions for claims about nested logical possibility as follows...

> *Let $\phi$ be a formula with no free variables. $\Diamond_{R_1,\dots,R_m}\phi$ is true relative to a model $\mathscr{M}$ just if there is another model $\mathscr{M}'$ which assigns the same sets of tuples to the extensions of $R_1, \dots, R_m$ as $\mathscr{M}$ and makes $\phi$ true.*

$\phi$ is true full stop if it is true relative to the model/interpretation $\mathscr{M}$ which interprets all nonmodal vocabulary standardly.

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