Coincidence Avoidance and Formulating The Access Problem

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Abstract

In this paper I discuss a trivialization worry for Hartry Field's official formulation of the 'access problem' in philosophy of mathematics, which was pointed out by Øystein Linnebo (and has recently been made much of by David Enoch and Justin Clarke-Doane). I argue that Linnebo's attempted reformulation of the Benacerraf problem fails to block this worry, but that access worriers can better defend themselves by sticking closer to Hartry Field's initial, informal, characterization of the access problem in terms of (something like) general epistemic norms of coincidence avoidance.

1 Introduction

In 'Mathematical Truth' [2], Benacerraf presents a classic worry for realists about mathematical objects – what is sometimes called the access problem. He argues from a causal constraint on knowledge to the conclusion that if realism were true, human knowledge of mathematics would be impossible. Many philosophers have been deeply moved by *something about* this worry (and analogous concerns in related domains), even while rejecting the specific premises employed in Benacerraf's argument [7] [16]. However, a satisfactory formulation of this access worry has proved elusive.

In the first half of this paper, I develop Hartry Field's idea that we should understand access worries in terms of realists' commitment to a match between human beliefs and realist facts which cries out for explanation and (apparent) inability to give a suitable explanation. I argue that existing elaborations of this core idea (by Field and others) go astray by attempting to fix on a single fact about human reliability R, such that access worries can be identified with the inability to explain R from any more general premises which the realist believes.

Instead, I propose, Fieldian access worries are better understood as motivated by a general norm of ceterus paribus coincidence reduction. Specifically, I propose that a realist theory of some domain of investigation (such as mathematics or morals) faces an access problem to the extent that accepting this theory (appears to) commit one to positing any 'extra' coincidences, beyond those required by competing, less realist, approaches to the same domain.

Accordingly, we shouldn't expect to find any single reliability claim, such that merely explaining it suffices to banish access worries. And it is easy to see what goes wrong with certain contentious responses to earlier formulations of the access problem [6][4], which explain away one apparent extra coincidence involving human accuracy, which the realist seems committed to positing, by appealing to another.

Finally, I conclude by arguing that we don't need a conceptual analysis of the relevant notion of coincidence to state or even hope to solve philosophical disputes about access worries. And I note that a certain moral which is sometimes drawn from considering intuitively unsatisfying and trivial answers to the access problem. One can think of philosophers like Justin Clarke-Doane and David Enoch who use intuitively unsatisfying 'trivial' explanations of human accuracy about mathematical and/or moral facts to argue that there is no access problem for these facts as suggesting that (either) our coincidence-avoidance intuitions about which correlations involving necessary truths 'cry out for explanation' are deeply unreliable, or that (despite appearances) all such cries for explanation can be adequately answered by just by 'stapling together' two unrelated explanations for each half of the coincidence (as these trivializing explanations do). But I note that accepting either alternative would require junking an important and apparently fruitful part of current mathematical practice.

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2 Field's Formulation of the Access Problem and Trivialization Worries

2.1 Field's Formulation of the Access Problem

Let me begin by reviewing Hartry Field's approach to the access problem, and the trivialization worries which have arisen for it.

In *Realism, Mathematics and Modality*[8], Field suggests that we should think of the access problem for mathematical realists as arising from a challenge for the realist to, "explain how our beliefs about [mathematical objects] can so well reflect the facts about them" in some internally coherent fashion. He notes that, "[I]f it appears in principle impossible to explain this [match between our beliefs and reality], then that tends to undermine ... belief in mathematical entities, despite whatever reason we might have for believing in them." I will develop and defend this core proposal in what follows.

However, Field elaborates this core idea in a way which (I will suggest) raises concerns about triviality. He argues that we can only attribute mathematicians knowledge of a mathematical claim ϕ if the following reliability claim holds. (I will discuss different ways of understanding this reliability claim below).

R: Reliably, if mathematicians accept that 'ϕ', then ϕ.

Admittedly, a thinker could have significant *true mathematical beliefs* without this kind of reliability, but Field suggests that such a person would not qualify as having knowledge. For example, a 'lucky fool', who decides whether or not to believe mathematical statements on the basis of a coin toss and winds up with many true beliefs in this way, would (plausibly) not count as knowing these mathematical statements.

Thus, because the mathematical realist believes we have significant mathematical knowledge, she is plausibly committed to accepting the above reliability claim. But, Field suggests, it appears in principle impossible for the realist to give any satisfactory explanation for R. And this fact casts doubt on the truth of realism.

The resulting view has obvious appeal. It has been used (with some minor modifications) to articulate access worries concerning other domains like morals and metaphysical possibility[1]. Unlike Benacerraf's original access worry, Field's formulation does not depend on any contentious assumptions about causal constraints on knowledge. Furthermore, Field's formulation appears to reveal an internal tension within the realist's beliefs. It thereby vindicates the common intuition that access worries are different from (and more troubling than) mere skepticism[2].

I think Field is quite right that the mathematical realist faces strong epistemic pressure to explain R, and that dispelling the impression that they can't do so is a necessary condition for dissolving access worries. However, we will see that explaining R is plausibly not *sufficient* to answer access worries. So, although Field's core approach is right and his further argument highlights crucial issues, it would be a mistake to take the final step of identifying access worries with inability to explain R.

[1]Just as it seems mysterious that our mathematical intuitions match objective facts about (say) platonic mathematical objects or proof transcendent coherence facts, it can seem mysterious that our a priori intuitions about goodness, beauty or what Lewissian possible worlds exist match objectives facts.

[2]It's not just that the access worrier can't justify their mathematical beliefs from indubitable premises which the skeptic accepts, but that their account of human accuracy seems troubling from their own point of view.

2.2 Safety and the Trivialization Problem

To see why explaining R is (plausibly) insufficient to answer access worries, let's consider a few different ways of cashing out the reliability claim made by R.

One popular approach is to read R as demanding that our mathematical beliefs be 'safe' in the sense that[3] they could not have easily been wrong, i.e., mathematicians' belief forming methods would not have lead them to form false beliefs at any suitably close possible worlds.

Another possibility, which Field mentions as a fall-back option, is to drop counterfactual considerations and simply say that the *actual* abundance of true mathematical beliefs and lack of false mathematical beliefs is something which the realist owes us an explanation for.

But if one takes either of these approaches, then (as Øystein Linnebo[16] and Justin Clarke-Doane[5] have separately noted) it seems like one can 'trivially' explain the relevant form of reliability, using other premises which the realist accepts, as follows.

> TRIV: Mathematicians reliably believe truths because they reliably believe only those mathematical claims which can be proved in a certain formal system Σ (something like first order reasoning from the axioms of ZFC), and this formal system is (necessarily) truth preserving.

TRIV seems to explain the safety of our mathematical beliefs. For, as it's robustly the case that mathematicians form their mathematical beliefs by making inferences authorized by Σ, they will continue to form mathematical beliefs via Σ in all relevantly close possible worlds. And the sentences derivable via Σ are all necessary truths. Hence all these close possible worlds will be ones in which they continue to form mostly true mathematical beliefs, thereby explaining the

[3]See Field's student Clarke-Doane's development of reliability in [3] and [4].

safety of our mathematical beliefs (as the realist understands them). And TRIV also seems to explain (at least in some sense) our possession of many true and few false mathematical beliefs (in the actual world), by pointing out that we arrive at mathematical beliefs by reasoning validly from true axioms.

However, it is equally clear that citing TRIV does nothing to satisfy intuitive access worries. This suggests that intuitive access worries cannot be reduced to the need to explain either the safety of our mathematical beliefs or the fact that we have many true and few false mathematical beliefs.

Now Field could, obviously, respond to this objection by denying that TRIV constitutes a genuine explanation for R (or for our possession of many true and few false mathematical beliefs) [8]. And this idea has some prima facie attraction.

However, many readers (like Linnebo and Clarke-Doane) seem to have the opposite intuition. And I think it is ultimately hard to deny that TRIV provides *some kind* of an explanation of R. For we can easily imagine non-philosophical contexts where TRIV would constitute an excellent response to an explanatory demand: an anthropologist could explain why some newly-discovered community is reliable about mathematics/had so many true and so few false mathematical beliefs by showing that all their mathematical reasoning can be reconstructed in terms of some formal system and then noting that this system is sound[4]. So, in the absence of further sharpening of the intuitive notion of explanation (something Field doesn't provide), it appears that TRIV does explain R and access worries cannot be reduced to the need to explain R.

Note, also, that one cannot defend Field's account of the access problem by rejecting TRIV merely on the grounds that it assumes the theorems of Σ are true in a realist sense (e.g., correctly describe the platonic objects) which

[4]Such explanation would admittedly be partial, but that doesn't prevent it from being an explanation. As David Lewis notes everything we give is a partial explanation: the accident occurred because of the bald tire, because of the driver's slipshod maintenance etc[14].

philosophers pressing an access worry wouldn't accept. For, this doesn't prevent TRIV from being an internally coherent explanation from the realist's perspective of our accuracy about mathematics. One of the great benefits of Field's proposal was that it appeared to reveal an *internal* problem for realism, not just a skeptical worry. Thus, it suffices for the realist to give an internally coherent explanation.

2.3 Interpreting R More Demandingly

One natural thought is to interpret the 'reliability' invoked in Field's R more demandingly, and use this as a basis for rejecting TRIV.

Suppose we grant that TRIV explains why there aren't any *extremely close* possible words at which mathematicians' beliefs are massively false. If we read Field's reliability claim R more demandingly – as requiring mathematicians to be accurate in a larger sphere of close possible words *including some where they don't form beliefs via* Σ – then we can still resist the claim that TRIV explains R[5].

It's not immediately obvious that the realist is committed to the truth of such a demanding version of R. Rigorously defending this approach would require arguing that the realist is committed to some specific and much higher degree of reliability, and I haven't seen anyone do this[6].

But I won't dwell on this hurdle, as I think a deeper problem is lurking. The problem is that we can imagine discoveries which would imply and (in a sense) explain *even very modally robust* agreement between human psychology and realist facts about something like math or morals, while still leaving intuitive access worries untouched. Thus, a more demanding interpretation of R is incapable of rescuing Field's elaboration of his core intuitions.

[5]This corresponds to individuating our methods more broadly

[6]This is a version of the famous 'generality problem' for reliablist epistemologies [9].

For example, consider the classic moral realist, who takes our beliefs about permissible favoritism toward relatives to be 'robustly objectively correct' in a sense which implies that creatures apparently inclined to advocate and practice a different degree of favoritism would have false beliefs about morality (rather than true beliefs about some equally good other notion shmorality of equal metaphysical status). Moral realists of this stripe intuitively face an access worry about the accuracy of our moral beliefs[7]. Now imagine such a moral realist attempting to address access worries by giving the following kind of explanation of our accuracy about permissible favoritism facts.

> EV-MOR: It is a robust fact that, in all circumstances conducive to the evolution of intelligence, natural selection favors the trait of advocating and valuing being twice as generous with immediate family as with other individuals. Furthermore, it is morally correct to be (exactly) twice as generous with family, and this is a necessary truth.

This story certainly seems to provide some kind of explanation for our accuracy about moral facts in a very wide range of possible worlds. Yet, considering it does nothing to answer intuitive access worries[8]. This is not just because the genealogy of morals suggested above is probably false. For even if imagine that that the evolutionary/game theory part of EV-MOR were unquestionably true and getting at a deeply reliable law of nature, considering EV-MOR would still do nothing to address intuitive access worries. Thus, Field's diagnosis of the access problem can't be rescued by increasing the level of reliability required.

[7] Note that even imperfect moral accuracy (at a rate substantially better than chance) can give rise to such an access worry.

[8] A similarly unsatisfying example explanation can be developed in the case of mathematics.

EV-MATH The only way for intelligence to evolve involves having a compositional language, and the only way that mathematics-like practices ever arise if from some fluke reusing the brain structures which compute grammaticality to produce assertions about certain mathematical structures, and it just so happens that these correspond to the platonic mathematical objects which actually exist.

2.4 Sensitivity and Counter-Possible Conditionals

A different strategy for understanding the reliability claim in Field's R is to appeal to metaphysically impossible worlds.

Employing metaphysically impossible worlds has little effect on safety[9]. However, it does give teeth to sensitivity requirements (another popular way of thinking about reliability). Sensitivity demands that if ϕ hadn't been true we wouldn't have believed ϕ (i.e., in the closest possible worlds where ϕ isn't true we don't believe ϕ). Our mathematical beliefs are trivially sensitive if we interpret this requirement using regular Lewissian counterfactuals (because there are no possible worlds where they are false).

However, demanding that realists explain sensitivity at metaphysically impossible worlds promises to let us reject explanations like TRIV and EV-MOR. For the fact that mathematicians reliably tend to accept propositions derivable from certain necessarily true axioms doesn't appear to explain why, in metaphysically impossible worlds where these axioms are false, we would still wind up having true mathematical beliefs. Indeed, such explanations seem to suggest that if mathematics/morals had been different then our beliefs would have been just the same (because these beliefs are shaped by unrelated evolutionary/game theoretic/anatomical considerations).

However, this approach actually faces very serious problems. First, there are reasons for doubting that we have any coherent shared grip on the closeness relation for metaphysically impossible scenarios (aka 'counterpossible conditionals'). For example, if $2 + 2 = 5$ would $+$ still satisfy the usual inductive definition? If not, how would things be different? Despite advances in understanding the

[9] As Justin Clarke-Doane points out, even if we allow that 'impossible worlds' where mathematical facts are different can in principle be relevant to truth conditions for counterfactuals, it would seem that these worlds would be very remote from the actual one – so it's not clear why explaining reliability should require showing that mathematicians' beliefs would continue to express truths in these very remote possible worlds[4].

logic of counterpossible conditionals [17], we still face significant uncertainty (or perhaps conceptual underdetermination) concerning the substantive closeness relation on impossible worlds[10]. Given this uncertainty, cashing out informal access worries in terms of a demand to explain counterpossible conditionals doesn't seem very helpful.

A second problem for this approach is that the counter-possible sensitivity requirement seems to fail (or is at least hard to explain) in many cases which are intuitively unproblematic. For example, if bachelors were unmarred women rather than unmarried men, would we still believe that bachelors are unmarried men[11]? Presumably there is no reason to doubt our knowledge of bachelorhood facts and this calls into question this interpretation of the sensitivity requirement above.

3 Linnebo and Alternative Languages

Now let us turn to Øysten Linnebo's attempt to fix Field's criterion. As noted above, Linnebo presses a version of the trivialization problem against Field. So one might hope that his version of the access problem would avoid trivialization worries. However, I will argue that it does not.

Linnebo accepts Field's supposition that access worries arise from the realist's inability to explain some particular fact. But he replaces Field's explanandum with a different reliability claim:

> R_{MS} If mathematical sentences (like "$2+2=4$") had not expressed truths then mathematicians wouldn't have accepted them.

[10] Or the substantive closeness relation which would be relevant to this attempt to formulate access worries, if there is some kind of context dependence as David Lewis has suggested[15].

[11] Justin Clarke-Doane gives a somewhat more complicated example along these lines in [4]: if the facts about what configurations of matter constituted a chair were different, would our beliefs be different?

In terms of possible worlds, R_{MS} asserts that, the closest possible worlds in which linguistic differences mean that the symbols/sounds "$2 + 2 = 4$" express a falsehood are ones in which mathematicians no longer accept this *sentence*[12]. Thus, one can think of Linnebo as using counterfactuals about semantic facts instead of *metaphysically impossible worlds* to spell out the sensitivity requirement from the prior section.

However, Linnebo's story faces its own trivialization problem, as well as an over-demandingness problem. To see this trivialization problem, note that it might easily be that the closest possible worlds where a mathematical sentence like "$2 + 2 = 4$" doesn't express a truth are ones where some superficial and recent change in language/orthography went differently, e.g., worlds where the transition from Roman numerals to Arabic numerals went differently so that the symbol "2" is now used to mean 3. These will indeed be worlds where people no longer accept "$2 + 2 = 4$" (since these are worlds in which the translation from Roman numerals went differently [13]). So it seems like we can explain Linnebo's R_{MS} by appealing to the possibility of orthographic change and our inclination to deny false mathematical claims.

More generally, Linnebo's R_{MS} conditional seems to be potentially explicable via all the unsatisfying explanations for human accuracy about mathematical/moral facts discussed in the previous section. For instance EV-MOR asserted that evolution and game theory determine that intelligent creatures are overwhelmingly likely to treat a certain amount of favoritism as permissible, and that ratio of permissible favoritism also happens to be objectively correct. Assuming EV-MOR is true, the closest worlds where, "helping friends twice as much as strangers is permissible" expresses a falsehood would plausibly be ones where our language is different rather than our moral sentiments being different,

[12]So, for example, mathematicians in this world don't assent to this sentence in conversation or place it into textbooks.

[13]Thanks to Warren Goldfarb, in conversation, for the example involving Roman numerals.

so that we don't accept this sentence (and Linnebo's metasemantic version of the sensitivity requirement is satisfied). Thus, if EV-MOR were true it would plausibly explains R_{MS} (as well as Field's R) without answering intuitive access worries.

This caveat raises the issue of what kind of grip we have on these linguistic counterfactuals at all. For example, if 'there are dogs' had expressed a falsehood, what claim would it have expressed? Would it still have expressed a true claim? There are many different scenarios where some sequence of symbols like "$2+2=4$" fails to express a truth and it's not at all clear that the closest such worlds are ones in which "$2+2=4$" even has anything to do with mathematics. This brings us to a second problem for Linnebo.

The problem is that we can construct cases where some quirk of history ensures the falsehood of the counterfactual R_{MS} in a way that does nothing to generate an intuitive access worry or any kind of problem with positing knowledge. For example, it's been argued that medieval science often expected deep analogies between different domains, so that very different things (personality types, metals, planets, mythical Greek gods) that somehow participated in the nature of Neputune would behave analogously. Imagine a possible world where analogous theories were developed for astrology and fledgling chemistry (and each had a special notation), so that there was a fairly simple correspondence between sentences expressing (supposed) truths of the astrological theory and those expressing (supposed) truths of the chemical theory in the year 800 CE. Now suppose that, because of these analogies, some monastic copying error swapped the symbols used to express chemical reactions and astrological claims so that "$H^+ + OH^- \rightleftharpoons H_2O$" went from originally expressing an astrological claim (say, the proposition that male Leos and female Libras are romantically linked when Mars is entering Scorpio) to expressing the claim that it expresses in normal English. And suppose that chemistry and astrology developed separately in the years after 800, with both continuing to enjoy great popularity. We can imagine a chemist who has (intuitively) justified beliefs about chemistry, and unjustified beliefs about astrology. Plausibly, some of the closest possible worlds to this one where "$H^+ + OH^- \rightleftharpoons H_2O$" fails to express a truth would be ones where this copying error never happened (rather than the very remote ones in which the chemical reactions proceed differently). In such worlds, the above sentence will express a widespread and a long standing, but false, doctrine about astronomy, which our horoscope-reading chemist also accepts. Thus, it won't be the case that, had various chemical sentences not expressed a truth, she wouldn't have believed them. Yet intuitively our chemist could qualify as having chemical propositions actually expressed by these sentences. Thus we seem to have a counterexample to R_{MS}.

This final problem is only heightened if we try to avoid trivializing explanations (like the roman numerals example discussed above) by strengthening our reliability requirements. For doing this only increases the risk of demanding too much, i.e., that Linnebo's conditional R_{MS} could fail for reasons (like the chancy chemical-astrological symbol swap) that do nothing to impugn our claims to knowledge of a given domain. Thus, there's no plausible interpretation of Linnebo's R_{MS} which lets us avoid both trivialization worries and appeal to a sensitivity principle which we have independent reason for doubting.

4 A Coincidence Avoidance Approach to Access Worries

4.1 Field's Core Idea and General Norms of Coincidence Avoidence

In view of the problems for spelling out (or replacing) Field's explanandum R discussed above, I propose that we stick to Field's initial characterization of the access problem in terms of general norms of coincidence avoidance – rather than trying to specify any single reliability fact, such that merely explaining this fact (from more general premises which the realist believes) would suffices to answer access worries.

Specifically, I propose that a realist theory of some domain of investigation (such as mathematics or morals) faces an access problem to the extent that accepting it commits one to positing a certain kind of coincidental match between human beliefs and the facts about that domain, but prevents one from giving any explanation which would remove this appearance of coincidence. A little more formally, a realist theory faces an access problem to the extent that:

> Combining this theory with uncontroversial claims about the extent of human accuracy about the domain in question forces us to posit *some* coincidental match between human beliefs and belief-independent facts which intuitively 'cries out for explanation.' However, no satisfactory explanation of this match is possible.

When this holds, it would seem that we have a significant (if defeasable) reason to reject the realist theory in question. Such theories are ceterus paribus undesirable, in that they commit us to positing an extra inexplicable coincidence: a match between human psychology and the realist's subject matter which cries out for explanation but cannot be explained.

Note that this constitutes an internal problem for the relevant theory. For the shared norms of coincidence avoidance which we draw on in phrasing access worries are *themselves* part of the realist's total picture of reality. Thus, (we can continue to say that) the realist faces an internal tension – in this case, a tension between their philosophical beliefs about some domain and their own sense of which kinds of correlations constitute an unattractive coincidence.

Also note that, on the view I'm advocating, access worries only give us ceterus paribus reason to reject a given realist theory of some domain. If it turns out that all the alternative views which avoid this access problem have worse flaws (as, e.g., formalist theories which have trouble capturing proof transcendent truth conditions and the role of math in the sciences plausibly do), this bullet might be worth biting.

While, strictly speaking, a theory has an access problem to the extent no satisfactory explanation of the match between beliefs and belief-independent facts is possible, I will sometimes speak loosely and say that a theory faces an access problem when it *appears* that no such explanation is possible (though, to be pedantic, it only apparently faces an access problem). When it no longer appears that no such satisfactory explanation is possible, I will say that the access problem has been solved or dissolved. Thus, classical attempts to eliminate access worries like Modal-Structuralism, Quantifier Variance, Quineinism and Neo-Fregean view can be seen as attempts to solve (or partially solve) the access problem as conceptualized above[14].

[14]Modal-Structuralism, Quantifier Variance, and Neo-Fregeanism help answer access worries (as characterized above) by suggesting that any logically coherent mathematical posits would express truths and thus explaining how any coherent mathematical beliefs we have correspond to mathematical truths (of course the issue of how we come to have coherent mathematical beliefs remains).

4.2 Helpful Consequences

Formulating Field's access worry as an application of more general norms of coincidence avoidance has two interesting and helpful consequences.

First, this proposal identifies access worries with a holistic problem with the realist's account and thus explains why (as noted above) they can't be dismissed by explaining one type of accuracy in terms of another, equally mysterious, type of accuracy.

For, on the view above (dis)solving one's access problem requires removing the appearance that one is committed to positing *any* extra coincidences. So we can allow that TRIV and EV-MOR do, in some sense, explain human possession of true beliefs but still maintain that they are useless in addressing access worries because each makes salient appeal to an extra coincidence, which more deflationary rival understandings of mathematical/moral practice let us avoid. Specifically, TRIV explains our accuracy about realist mathematical facts by appealing to an unexplained coincidental-seeming match between the mathematical reasoning method Σ and realist mathematical facts. And EV-MOR only explains our good intuitions about morality by appealing to an unexplained match between game theoretic optimality and objective moral facts.

Second, this approach suggests an important way in which access worries can be a matter of degree. While a philosophical theory either does or doesn't allow for an explanation of R or R_{MS} (and thus does or doesn't face an access problem), on this approach one theory can be preferred to another as it requires accepting fewer coincidences.

Because of this comparative element, we should *not* think of access worries as invoking an epistemic requirement to 'consign to the flames' every theory that posits a coincidence (analogous to Hume's famous empiricist exhortation to reject all concepts that weren't suitably related to experience[10]). Instead, access worriers appeal to general norms in favor of *reducing* the number of coincidences one is committed to positing, insofar as this is compatible with other epistemic goals.

This is important and helpful because it means that, even if our knowledge of inductive generalization raises an access problem on its own right (maybe even an insoluble access problem), we can still invoke inductive generalization to dispel our access worries regarding a domain like mathematics (as no rival theory would dispel the coincidence that the future seems to behave like the past). My view allows theories to suffer access worries to varying degrees depending on the number and implausibility of the coincidences they are committed to positing.

4.3 Do We Owe a Further Analysis of Coincidence?

This way of understanding access worries can seem to require using an unacceptably imprecise notion of coincidence avoidance. However, I want to point out that the same imprecise notion plays an important role in scientific and philosophical reasoning.

We clearly have a practice of distinguishing certain parts of a theory as unattractive coincidences. And we take commitment to any such extra coincidences to be a (ceterus paribus) reason to disfavor a theory. Think of the kind of argument we might use to convince someone to stop believing in the Loch Ness monster. We generally wouldn't be able to derive the non-existence of the monster from beliefs we share with the Loch Ness conspiracy theorist, or locate a literal contradiction within their beliefs. Rather, we would point out unattractive extra coincidences which the Loch Ness monster theory has to admit (the monster never shows up when someone has a really good camera, it only appears in pictures which could plausibly be faked etc.) but can't elegantly explain. We appeal to a kind of shared general epistemic norm, which says that one has ceterus paribus reason to avoid theories which posit certain kinds of (inexplicable) coincidences. What results isn't a deduction that the Loch Ness monster doesn't exist, but rather, ceterus paribus reasons for disfavoring its existence.

Admittedly, what makes something a coincidence is rather complicated. Coincidences aren't just facts posited by a theory which would otherwise be assigned low probability given the rest of a theory. For example, any particular long sequence of outcomes of a coin toss is unlikely, but we don't take total theories of the world which include the results of past coin tosses to be committed to an extra unattractive coincidence[15]. Nonetheless, spotting and rejecting such coincidences plays an important role in scientific and commonsense reasoning[16] , even when we can't appeal to anything like a general Carnapian logic of induction. We might wish to have a tidy and uncontroversial criterion for when a theory counts as positing extra coincidences. However, we are all committed to using this kind of reasoning all the time, on a 'know it when you see it' basis. Thus, it seems reasonable to take these intuitions about theoretical badness at face value.

4.4 Coincidences Involving Necessary Truths

Now one might imagine the philosophers (such as Clarke-Doane [4] and David Enoch [6]) who have pressed trivializing responses to the access problem responding to my proposal as follows. They might allow the above general point about recognizing unsavory coincidences, but suggest (perhaps partly on the basis of mathematical access worriers' failure to cash out their intuitive appeals

[15] The feeling of coincidence/crying out for explanation seems related to an intuition that some other theory predicting the same things but with fewer dimensions of freedom should exist, but the question of a priori theory plausibility is an infamously hard one and I won't speculate about this more here.

[16] For example the clustering of the orbits of many trans-Neptunian objects has lead astronomers to hypothesize the existence of a 9th planet orbiting beyond 200 AU [19].

to coincidence avoidance in other terms) that some special special goes wrong when we apply these intuitions to evaluating whether mathematical realists face an access problem. Specifically, one can think of them as suggesting that (either) our coincidence-avoidance intuitions about which correlations involving necessary truths 'cry out for explanation' are deeply unreliable, or that (despite appearances) all such cries for explanation can be adequately answered by just by 'stapling together' two unrelated explanations for each half of the coincidence (as these trivializing explanations do).

However I think this line is hard to maintain. First, trivializers haven't presented much reason for thinking that analyzing the notion of coincidence avoidance in cases where both sides of the relevant coincidence are contingent truths is any easier. Attempts to analyze paradigmatic contingent cases of crying out for explanation in terms of equally questionable notions like that of extraordinary types[18]. In contrast, there are plenty of good paradigms for how to think about coincidence avoidance that would allow it to apply to both necessary and contingent regularities, e.g., in terms of preferring theories that have fewer degrees of freedom and a general scientific desideratum to prefer theories that unify [11].

Second, and more importantly, saying that our intuitions about coincidence avoidance become incoherent when applied to necessary truths seems to conflict with existing mathematical methodology, which does seem to use explanation-seeking and coincidence avoidance to guide research[1][13]. For example, the history of John Conway's 'Monsterous Moonshine' conjecture is provides a particularly dramatic illustration of how noting a striking relationship[17] between

[17] "Strangely enough, this function's first important coefficient is 196,884, which McKay instantly recognized as the sum of the monster's first two special dimensions.

Most mathematicians dismissed the finding as a fluke, since there was no reason to expect the monster and the j-function to be even remotely related. However, the connection caught the attention of John Thompson, a Fields medalist now at the University of Florida in Gainesville, who made an additional discovery. The j-function's second coefficient, $21,493,760$, is the sum of the first three special dimensions of the monster: $1 + 196,883 + 21,296,876$. It seemed as two pure mathematical facts and expecting a deeper unified explanation can lead to important discoveries *even when a proof of both facts already exists.* Similarly, in philosophy we seem happy to accept that avoiding coincidence in the sense of favoring theories that unify many explanations with few resources.

Proponents of the trivializing explanation haven't shown that there's any *principled and theoretically attractive line* which carves off the specific intuitions about coincidence avoidance and necessary truths which he wants us to be suspicious of (those driving access worries) from general methods of reasoning which are attractive and ubiquitous in philosophy and mathematics. Therefore, absent a stronger argument that such reasoning leads us astray, I don't see any reason to eschew its use.

4.5 A final note about tractability

Let me conclude by addressing a question about the tractability of disputes concerning access problems. Many philosophers currently disagree about how much of an access worry various forms of realism about mathematics, morals etc. face.

In this paper I have argued that critics of mathematical/moral realism can reasonably articulate and press an access worry by appealing to shared intuitive norms of coincidence avoidance, while taking a 'know it when we see it' attitude to the relevant concept of coincidence, rather than providing any explicit theory of what it takes for something to be an unattractive coincidence.

One might fear that adopting this position makes disputes about the access problem deeply intractable – by letting access worriers issue their challenge from an unassailable swampland of brute intuitions, without committing themselves to any general theses which the realist could defend themselves by attacking.

if the j-function was somehow controlling the structure of the elusive monster group."[12]

However I will argue that such pessimism is unwarranted, as there are other credible ways in which debate about access worries can be carried out, and by which widespread philosophical agreement could plausibly be produced.

On one hand, realists can reasonably hope to win over opponents , by providing an suitable *sample explanation* for our accuracy about the relevant domain which suffices to intuitive to banish coincidences (or only employs coincidences which anti-realists about the relevant domain are independently committed to positing). For instance, consider the very rough story provided by Quinean empiricism about mathematics[18]. On the Quinean picture, we acquire largely true beliefs about mathematics in the course of acquiring accurate scientific beliefs as a whole – by adopting empirically motivated scientific theories which force us to quantify over suitable mathematical objects.

There are well known problems for this Quinean story. For example, we seem to get mathematics right in advance of scientific applications, and we seem to know about the existence of mathematical objects in areas such as higher set theory which may forever lack relevant scientific applications). But it illustrates the kind of approach that could alleviate access worries (as opposed to merely silencing them and inspiring worriers to find a new form to articulate their discomfort[19]). For (unlike the trivializing explanations EV-MOR and TRIV criticized above), Quine's story appears to avoid appealing to any extra unattractive coincidences which the mathematical anti-realist can avoid positing[20].

Thus, if Quine's story worked, it would explain accuracy about mathematics using only faculties which his anti-realist opponents have independent reason for

[18]Here I will be discussing Quinean Empiricism understood as a story about the actual history of how human beings got accurate beliefs about mathematics, rather than (as I take Quine to have intended it to be) a story about how our beliefs our justified.

[19]Consider the way that standard objections to the Benacerraf's original formulation of the access problem via rejection of his causal constraint on knowledge felt unsatisfying.

[20]Arguably Quine's story employs *some* unattractive unexplained coincidences, by presuming that human faculties of doing scientific induction and inference to the best explanation are generally accurate. But this is not a problem, because mathematical anti-realists already accept such claims.

accepting as reliable. Accordingly it provides a kind of explanation which could dispel intuitive access worries for (ontological) realism about mathematics, by showing that appearances that accepting realism about mathematical objects commits one to positing *extra* coincidences (i.e., ones which could be avoided just by adopting a more deflationary philosophy of mathematics) are illusory.

Conversely, there are also credible paths to philosophical agreement that there is a genuine access problem for realism about a given domain. For example, a history of massive effort and continued failure to discover any plausible explanation of a certain coincidence can itself gradually increase access worries on my account. Thus, this way of formulating the access problem account provides a way for access worries to get worse and a way for them to get better.

5 Conclusion

In this paper I discussed trivialization worries for Hartry Field's formal characterization of the access problem, and noted some problems with existing responses to this trivialization worry (such as the one proposed by Øysten Linnebo in [16]).

I argued that access worriers can avoid these problems by sticking to Field's initial characterization of the access problem in terms of shared general norms of coincidence avoidance/reduction – and then denying that they owe any specific account of what it takes for something to be a coincidence. Because of the good work which this notion of coincidence reduction already does in mathematics and the sciences, it is something all parties in debate are committed to. Moreover, I've argued that the resulting picture does not render access worries intractable as might be feared in the absence of a more precisely spelled out notion of what an access worry consists of.

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