

Hubris, Coincidence Avoidance and the Access Problem

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Abstract

In this paper I discuss a trivialization worry for currently popular formulations of the ‘access problem’ in philosophy of mathematics. I argue that we can avoid this worry by relating access worries to general epistemic norms of coincidence avoidance. Specifically, I propose that a realist theory of some domain of investigation (such as mathematics or morals) *faces an access problem* to the extent that accepting this theory commits one to positing *any* extra unexplained coincidence beyond those required by competing deflationary approaches to the same domain. I then use this formulation of the access problem to diagnose what goes wrong in Justin Clarke-Doane’s recent argument that there can be no access problem.

1 Introduction

In ‘Mathematical Truth’ [2], Benacerraf presents an influential problem for realism about mathematical objects – what is sometimes called the access problem. He argues from a causal constraint on knowledge to the conclusion that if realism were true, human knowledge of mathematics would be impossible. Many philosophers have been deeply moved by *something about* this worry (and analogous concerns in related domains), even while rejecting the specific premises employed in Benacerraf’s argument [5] [14]. However, a satisfactory formulation of this access worry has proved elusive.

In this paper, I will attempt to provide such a formulation. In doing so, I will draw on insights from Hartry Field and Øysten Linnebo. I argue that Field and Linnebo were right to think of access worries as arising from realists’

commitment to a match between human beliefs and realist facts which *in some sense* cries out for explanation, plus their inability to give a suitable explanation. However, I will argue that their attempts to develop this story in more detail went astray; by formulating access worries too narrowly they opened the door to certain trivializing responses.

I will suggest that access worries are best understood as arising from a general norm of (*ceteris paribus*) coincidence reduction. In contrast to previous formulators of the access problem, I reject the idea that there's any particular reliability claim, such that any (internally coherent, realist) explanation of it would suffice to dispel the access problem. Instead, I propose that a realist theory of some domain of investigation (such as mathematics or morals) faces an access problem to the extent that accepting this theory (appears to) commit one to positing *any* extra unexplained coincidence beyond those required by competing approaches to the same domain. Moreover, I argue that no further precisification of this notion of coincidence is necessary to evaluate whether a theory faces an access problem. This approach lets us avoid trivializing responses to earlier formulations which (as we will see) explain away one apparent coincidence involving human accuracy by positing another such coincidence.

In the first half of this paper, I will review some problems for Field's formulation of the access problem and related approaches (such as Linnebo's). In the second half, I will develop my positive proposal, and put it in a larger epistemological and philosophical context.

I use my formulation of the access problem to directly answer Justin Clarke-Doane's influential argument that there can be no access problem in [4]. And I further argue that accepting a certain moral which is often drawn from Justin Clarke-Doane's arguments – that (either) coincidence-avoidance intuitions involving necessary truths are somehow deeply unreliable or that the appearance

of an unattractive coincidences can be explained away just by ‘stapling together’ two unrelated explanations for each half of the coincidence – would have manifestly unacceptable consequences for the practice of mathematics.

I admit that it would be nice to have a reductive analysis of the notion of ‘a priori unattractive coincidence’ employed my (intentionally) straightforward and non-reductive formulation of the access problem. But philosophers have famously failed to give a satisfying conceptual analysis of ‘chair’ for hundreds of years, and coincidence avoidance intuitions about necessary domains do important work within mathematics. So, I argue, it would be absurd hubris to dismiss these intuitions (or the access worry which they give rise to) merely on the grounds that philosophers haven’t produced a satisfyingly reductive analysis of the relevant notion of coincidence yet.

2 Field’s Formulation of the Access Problem and Trivialization Worries

2.1 Field’s Formulation of the Access Problem

Let me begin by reviewing Hartry Field’s approach to the access problem, and the trivialization worries which have arisen for it.

In *Realism, Mathematics and Modality*[6], Hartry Field develops an influential formulation of the access problem. He suggests that we should think of the access problem for mathematical realists as arising from a challenge for the realist to, “explain how our beliefs about [mathematical objects] can so well reflect the facts about them” in some internally coherent fashion. He notes that, “[I]f it appears in principle impossible to explain this [match between our beliefs and reality], then that tends to undermine ... belief in mathematical entities, despite whatever reason we might have for believing in them.”

Field develops this initial idea as follows. He argues that mathematicians can only have mathematical knowledge if the following reliability claim is true for most mathematical sentences ϕ (I will discuss different ways of cashing out this reliability claim in detail below).

R: Reliably, if mathematicians accept that ' ϕ ', then ϕ .

Admittedly, a thinker could have significant *true mathematical beliefs* without this kind of reliability, but Field suggests that such a person would not qualify as having knowledge. For example, a 'lucky fool', who decides whether or not to believe mathematical statements on the basis of a coin toss and winds up with many true beliefs in this way, would (plausibly) not count as knowing these mathematical statements.

Thus, because the mathematical realist believes we have significant mathematical knowledge, she is committed to accepting the above reliability claim. But it appears in principle impossible for the realist to give any satisfactory explanation for R and this casts doubt on the truth of realism.

Field's approach to access worries has obvious appeal. It has been used (with some minor modifications) as a way of articulating access worries concerning other domains like morals and metaphysical possibility¹. Unlike Benacerraf's original access worry, Field's formulation does not depend on any contentious assumptions about causal constraints on knowledge. Furthermore, Field's formulation appears to reveal an internal tension within the realist's beliefs. It thereby vindicates the common intuition that access worries are different from (and more troubling than) mere skepticism².

¹Just as it seems mysterious that our mathematical intuitions match objective facts about (say) platonic mathematical objects or proof transcendent coherence facts, it can seem mysterious that our a priori intuitions about goodness, beauty or what causally isolated Lewisian possible worlds exist match objectives facts.

²It's not just that the access worrier can't justify their mathematical beliefs from indubitable premises which the skeptic accepts, but that their account of human accuracy seems troubling only from their own point of view.

I think Field is quite right that in order to address their access worries, the realist must explain R. However, (as noted above) I will suggest that he errs in suggesting that merely providing an internally coherent explanation for R *suffices* to answer access worries.

2.2 Safety and the Trivialization Problem

To see what the problem is, let us consider a few different ways of fleshing out the reliability claim in R.

One popular approach is to read R as demanding that our mathematical beliefs be safe in the sense that³ they could not have easily been wrong, i.e., mathematicians' belief forming methods would not have lead one to form false beliefs at any suitably close possible worlds.

Another possibility, which Field mentions as a fall-back option, is to drop reliability and simply say that the actual abundance of true mathematical beliefs and lack of false mathematical beliefs is something which the realist owes us an explanation for.

But if one takes either of these approaches, it appears that (as Øystein Linnebo[14] and Justin Clarke-Doane[4] have separately noted) one can trivially explain Field's reliability claim R, using other premises which the realist accepts, as follows.

TRIV: Mathematicians reliably believe truths because they reliably believe only those mathematical claims which can be proved in a certain formal system Σ (something like first order reasoning from the axioms of ZFC), and this formal system is (necessarily) truth preserving.

TRIV seems to explain both the truth of our mathematical beliefs, and

³See Field's student Clarke-Doane's development of reliability in [3] and [4]

their safety, as follows. As it's robustly the case that mathematicians form their mathematical beliefs by making inferences authorized by Σ , they will continue to form mathematical beliefs via Σ in all relevantly close possible worlds. But the sentences derivable via Σ are all necessary truths. Hence all these close possible worlds will be ones in which they continue to form mostly true mathematical beliefs, thereby explaining the safety of our mathematical beliefs (as the realist understands them).

However, it is equally clear that citing TRIV does nothing to satisfy intuitive access worries. This suggests that intuitive access worries cannot be reduced to the need to explain Field's R.

Field would presumably deny that TRIV constitutes a genuine explanation for R [6]. But many readers (like Linnebo and Clarke Doane) have the opposite intuition. And I think it is hard to deny that TRIV provides *some kind* of an explanation of R. For we can easily imagine non-philosophical contexts where TRIV would constitute an excellent response to an explanatory demand: an anthropologist could explain why some newly-discovered community is reliable about mathematics by showing that all their mathematical reasoning can be reconstructed in terms of some formal system and then noting that this system is sound⁴. So, in the absence of further sharpening of the intuitive notion of explanation (something Field doesn't provide), it appears that TRIV does explain and access worries cannot be reduced to the need to explain R.

Note, also, that one cannot defend Field's account of the access problem by rejecting TRIV merely on the grounds that it assumes the theorems of Σ are true in a realist sense (e.g., correctly describe the platonic objects) which philosophers pressing an access worry wouldn't accept. For, this doesn't prevent TRIV from being an internally coherent explanation from the realist's perspec-

⁴Such explanation would admittedly be partial, but that doesn't prevent it from being an explanation. As David Lewis notes everything we give is a partial explanation: the accident occurred because of the bald tire, because of the driver's slipshot maintenance etc[12].

tive of our accuracy about mathematics. One of the great benefits of Field's proposal was that it appeared to reveal an *internal* problem for realism, not just a skeptical worry. Thus, it is important to remember that the realist is only being called upon to provide an internally coherent explanation.

2.3 Interpreting R more demandingly

One natural thought is to interpret the 'reliability' invoked in Field's R more demandingly, and use this as a basis for rejecting TRIV.

Suppose we grant that TRIV explains why there aren't any *extremely close* possible words at which mathematicians' beliefs are massively false. If we read Field's reliability claim R more demandingly – as requiring mathematicians to be accurate in a larger sphere of close possible words *including some where they don't form beliefs via Σ* – then we can still resist the claim that TRIV explains R⁵.

It's not immediately obvious that the realist is committed to the truth of such a demanding version of R. Rigorously defending this approach would require arguing that the realist is committed to some specific and much higher degree of reliability, and I haven't seen anyone do this⁶.

But I won't dwell on this hurdle, as I think a deeper problem is lurking. The problem is that we can imagine discoveries which would imply and (in a sense) explain *even very modally robust* agreement between human psychology and realist facts about something like math or morals, while still leaving intuitive access worries untouched. Thus, a more demanding interpretation of R is incapable of rescuing Field's proposal.

For example, consider the classic moral realist, who takes our beliefs about permissible favoritism toward relatives to be 'robustly objectively correct' in a

⁵This corresponds to individuating our methods more broadly

⁶This is a version of the famous 'generality problem' for reliabilist epistemologies [7].

sense which implies that creatures apparently inclined to advocate and practice a different degree of favoritism would have false beliefs about morality (rather than true beliefs about some equally good other notion of morality of equal metaphysical status). Moral realists of this stripe intuitively face an access worry about the accuracy of our moral beliefs⁷. Now imagine such a moral realist attempting to address access worries by giving the following kind of (obviously cartoonish) explanation of our accuracy about permissible favoritism facts.

EV-MOR: It is a robust fact that, in all circumstances conducive to the evolution of intelligence, natural selection favors the trait of advocating and valuing being twice as generous with immediate family as with other individuals. Furthermore, it is morally correct to be (exactly) twice as generous with family, and this is a necessary truth.

This story certainly seems to provide some kind of explanation for our accuracy about moral facts in a very wide range of possible worlds. Yet, this explanation does nothing to answer intuitive access worries⁸. Thus, Field's diagnosis of the access problem can't be rescued by increasing the level of reliability required.

2.4 Sensitivity and Counter-Possible Conditionals

A different strategy for understanding the reliability claim in Field's R is to appeal to metaphysically impossible worlds.

⁷Note that even imperfect moral accuracy (at a rate substantially better than chance) can give rise to such an access worry.

⁸A similarly unsatisfying example explanation can be developed in the case of mathematics. EV-MATH The only way for intelligence to evolve involves having a compositional language, and the only way that mathematics-like practices ever arise is from some fluke reusing the brain structures which compute grammaticality to produce assertions about certain mathematical structures, and it just so happens that these correspond to the platonic mathematical objects which actually exist.

Employing metaphysically impossible worlds has little effect on safety⁹. However, it does give teeth to sensitivity requirements (another popular way of thinking about reliability). Sensitivity demands that if ϕ hadn't been true we wouldn't have believed ϕ (i.e., in the closest possible worlds where P isn't true we don't believe P). Our mathematical beliefs are trivially sensitive if we interpret this requirement using regular Lewisian counterfactuals (because there are no possible worlds where they are false).

However, demanding that realists explain sensitivity at metaphysically impossible worlds promises to let us reject explanations like TRIV and EV-MOR. For the fact that mathematicians reliably tend to accept propositions derivable from certain necessarily true axioms doesn't appear to explain why, in metaphysically impossible worlds where these axioms are false, we would still wind up having true mathematical beliefs. Indeed, such explanations seem to suggest that if mathematics/morals had been different then our beliefs would have been just the same (because these beliefs are shaped by unrelated evolutionary/game theoretic/anatomical considerations).

However, this approach actually faces very serious problems. First, there are reasons for doubting that we have any coherent shared grip on the closeness relation for metaphysically impossible scenarios (aka 'counterpossible conditionals'). For example, if $2 + 2 = 5$ would $+$ still satisfy the usual inductive definition? If not, how would things be different? Despite advances in understanding the logic of counterpossible conditionals [15], we still face significant uncertainty (or perhaps conceptual underdetermination) concerning the substantive closeness relation on impossible worlds¹⁰. Given this uncertainty, cashing out informal

⁹As Justin Clarke-Doane points out, even if we allow that 'impossible worlds' where mathematical facts are different can in principle be relevant to truth conditions for counterfactuals, it would seem that these worlds would be very remote from the actual one – so it's not clear why explaining reliability should require showing that mathematicians' beliefs would continue to express truths in these very remote possible worlds[4].

¹⁰Or the substantive closeness relation which would be relevant to this attempt to formulate access worries, if there is some kind of context dependence as David Lewis has suggested[13].

access worries in terms of a demand to explain counterpossible conditionals doesn't seem very helpful.

A second problem for this approach is that the counter-possible sensitivity requirement seems to fail (or is at least hard to explain) in many cases which are intuitively unproblematic. For example, if bachelors were unmarried women rather than unmarried men, would we still believe that bachelors are unmarried men¹¹? Presumably there is no reason to doubt our knowledge of bachelorhood facts and this calls into question the sensitivity requirement above.

3 Linnebo and Alternative Languages

Now let us turn to Øysten Linnebo's attempt to fix Field's criterion. As noted above, Linnebo presses a version of the trivialization problem against Field. So one might hope that his version of the access problem would avoid trivialization worries. However, I will argue that does not.

Linnebo accepts Field's supposition that access worries arise from the realist's inability to explain some particular fact. But he replaces Field's explanandum with a different reliability claim:

R_{MS} If mathematical sentences (like " $2 + 2 = 4$ ") had not expressed truths then mathematicians wouldn't have accepted them.

In terms of possible worlds, R_{MS} asserts that, the closest possible worlds in which linguistic differences mean that the symbols/sounds " $2 + 2 = 4$ " express a falsehood are ones in which mathematicians no longer accept this *sentence*.¹².

Thus, one can think of Linnebo as using counterfactuals about semantic facts

¹¹Justin Clarke-Doane gives a somewhat more complicated example along these lines in [4]: if the facts about what configurations of matter constituted a chair were different, would our beliefs be different?

¹²So, for example, mathematicians in this world don't assent to this sentence in conversation or place it into textbooks.

instead of *metaphysically impossible worlds* to spell out the sensitivity requirement from the prior section.

However, Linnebo’s story faces its own trivialization problem, as well as an over-demandingness problem. To see this trivialization problem, note that it might easily be that the closest possible worlds where a mathematical sentence like “ $2 + 2 = 4$ ” doesn’t express a truth are ones where some superficial and recent change in language/orthography went differently, e.g., worlds where the transition from Roman numerals to Arabic numerals went differently so that the symbol “2” is now used to mean 3. These will indeed be worlds where people no longer accept “ $2 + 2 = 4$ ” (since these are worlds in which the translation from Roman numerals went differently¹³). So it seems like we can explain Linnebo’s R_{MS} by appealing to the possibility of orthographic change and our inclination to deny false mathematical claims.

More generally, Linnebo’s R_{MS} conditionals seems to be potentially explainable via all the unsatisfying explanations for human accuracy about mathematical/moral facts discussed in the previous section. For instance EV-MOR asserted that evolution and game theory determine that intelligent creatures are overwhelmingly likely to treat a certain amount of favoritism as permissible, and that ratio of permissible favoritism also happens to be objectively correct. Assuming EV-MOR is true, the closest worlds where “helping friends twice as much as strangers is permissible” expresses a falsehood would be ones where we don’t accept this sentence (so Linnebo’s metasemantic version of the sensitivity requirement is satisfied). At least, this explanation seems sufficient assuming that “permissible” continues to express a similar concept at close possible worlds. Thus, EV-MOR plausibly explains R_{MS} (as well as Field’s R) without answering access worries.

This caveat raises the issue of what kind of grip we have on these linguistic

¹³Thanks to ██████████, in conversation, for the example involving Roman numerals.

counterfactuals at all. For example, if ‘there are dogs’ had expressed a falsehood, what claim would it have expressed? Would it still have expressed a true claim? There are many different scenarios where some sequence of symbols like “ $2+2=4$ ” fails to express a truth and it’s not at all clear that the closest such worlds are ones in which “ $2+2=4$ ” even has anything to do with mathematics. This brings us to a second problem for Linnebo.

The problem is that we can construct cases where some quirk of history ensures the falsehood of the counterfactual R_{MS} in a way that does nothing to generate an intuitive access worry or any kind of problem with positing knowledge. For example, medieval science often expected deep analogies between different domains, so that very different things (personality types, metals, planets, mythical Greek gods) that somehow participated in the nature of Neptune would behave analogously. Imagine a possible world where analogous theories were developed for astrology and fledgling chemistry (and each had a special notation), so that there was a fairly simple correspondence between sentences expressing (supposed) truths of the astrological theory and those expressing (supposed) truths of the chemical theory. Now suppose that, because of these analogies, some monastic copying error swapped the symbols used to express chemical reactions and astrological claims so that “ $H^+ + OH^- \rightleftharpoons H_2O$ ” went from originally expressing an astrological claim (say, the proposition that male Leos and female Libras are romantically linked when Mars is entering Scorpio) to expressing the claim that it expresses in normal English. Plausibly, some of the closest possible worlds to this one where “ $H^+ + OH^- \rightleftharpoons H_2O$ ” fails to express truths would be ones where this copying error never happened (rather than ones in which the chemical reaction proceeds differently). In such a situation R_{MS} would be false, as we might very well accept the relevant astrological claim in such possible worlds without calling into question our chemical knowledge¹⁴.

¹⁴As unlikely events do happen, we can’t rely on the fact that in the *actual world* there

This overdemandingness problem is only heightened if we try to fix the problem of trivializing explanations by increasing the strength of the intended reliability claim. For doing this only increases the risk of demanding too much, i.e., that Linnebo’s conditional R_{MS} could fail for reasons (like the chancy chemical-astrological symbol swap) that do nothing to impugn our claims to knowledge of a given domain. Thus, there’s no plausible interpretation of Linnebo’s R_{MS} which lets us avoid both trivialization worries and appeal to a sensitivity principle which we have independent reason for doubting.

4 Explanationist Constraints on Knowledge

Another possible strategy for diagnosing what goes wrong with explanations like EV-MOR (arguably employed by Sharon Street [18]) is that, although they do explain the connection between human psychology and moral/mathematical facts, they fail to address access worries because they suggest that the truth of moral/mathematical propositions (as the realist understands them) plays no role in explaining why we are inclined to accept it.

To motivate this idea, consider the explanation EV-MOR given above. In this scenario, game-theoretic considerations explain why intelligence is unlikely to arise except in combination with a certain proclivity for nepotism. But the fact that these beliefs about permissible favoritism are *true* seems to play worryingly little role in the best explanation for why *we have them*.

We might try advocating an explanationist constraint on knowledge or justification along the following lines **Exp K**: “If S knows that P, then P must

wasn’t some very implausible copying error which miraculously swapped the chemical and astrological symbols (with the unlikely result that chemical assertions became understandable astrological assertions and vice-versa). If Linnebo’s diagnosis is correct then, in such a case, we should face an access problem in chemistry as worlds that didn’t undergo such a swap would be very close. However, deriving chemical knowledge from careful experiment isn’t epistemically assailable just because an unlikely event was responsible for how we write down this knowledge.

figure in the best explanation of S's belief that P", and use this to explicate a form of access worries as follows.

Realists about a certain domain face an access problem insofar as they take us to have knowledge of this domain but realist facts (i.e., the kind of facts the realist takes to provide the content for beliefs about this domain) do not figure in the best explanation of beliefs about this domain. Thus the fact that our best scientific explanations for our moral beliefs do not mention realist facts can be thought of as 'debunking' moral realism. And moral realists face a problem if realist facts about morality don't play a role in the best explanation for our possession of moral beliefs.

Can we use this idea to explicate intuitive access worries? I think we can't because Exp K is controversial and often rejected by people who feel strong intuitive access worries.

For one thing, Exp K doesn't seem to leave room for knowledge by stipulation or knowledge of the future. It's just not clear that the best explanation for my belief that James would be on time to class today involves the fact that he was on time to class today, rather than the fact that he was on time to class *in the past*, or some general principle implying that he tends to be on time. Or consider a case where I gain knowledge of a fact like 'all bachelors are men' by making a stipulative definition of the term "bachelor". Presumably, I can have knowledge as a result. Yet there's no obvious sense in which *the fact* that all bachelors are men would explain *my belief* that all bachelors are men. Thus I take there to be (prima face) independent reason to doubt the truth of Exp K.

While those who favor explanationist constraints on knowledge have attempted several (all controversial) rebuttals to these kinds of criticisms¹⁵, we

¹⁵Carrie Jenkins articulates a more sophisticated version of Explanationism, which gets around the problems above by appealing to a special explanatory context, rather than the best explanation. But she says very little about what this context is, so invoking it doesn't shed much light on access worries[9].

are still left with a version of the difficulty faced by Benacerraf’s original formulation of the access problem: we can articulate a concern for realism only by assuming a very contentious principle (e.g., a causal or explanationist constraint on knowledge). Unless we believe that the many people who feel a strong intuitive access worry are unknowingly relying on such a principle (despite being disposed to explicitly doubt or reject it), this seems like a poor analysis of the access problem.

5 Access Worries As Coincidence Avoidance

5.1 My Proposal

I will now articulate my own preferred understanding of the access problem. My story will adopt Field’s idea that access worries arise from the realist’s apparent inability to “explain how our beliefs about [mathematical objects] can so well reflect the facts about them,” in an internally coherent fashion. But it will develop this idea in a more abstract and demanding way than Field himself does, by appealing to general norms of coincidence avoidance.

Specifically, I propose that a realist theory of some domain of investigation (such as mathematics or morals) *faces an access problem* to the extent that accepting it commits one to positing a certain kind of *coincidental* match between human beliefs and the facts about that domain, but prevents one from giving any explanation which would remove this appearance of coincidence. A little more formally, a realist theory faces an access problem to the extent that:

Combining this theory with uncontroversial claims about the extent of human accuracy about the domain in question forces us to posit *some* coincidental match between human beliefs and belief-independent facts which intuitively ‘cries out for explanation.’ How-

ever, no satisfactory explanation of this match is possible.

When this holds, it would seem that we have a significant (if defeasable) reason to reject the realist theory in question. Such theories are *ceteris paribus* undesirable in that they commit us to positing an extra inexplicable coincidence: specifically, a match between human psychology and the realist's subject matter which intuitively cries out for explanation but cannot be explained.

Note that this constitutes an internal problem for the relevant theory. For the shared norms of coincidence avoidance which we draw on in phrasing access worries are *themselves* part of the realist's total picture of reality. Thus, (just as in Field's formulation) the realist faces an internal tension – between their philosophical beliefs about some domain and their own sense of which kinds of correlations constitute an unattractive coincidence.

Also access worries only give us *ceteris paribus* reason to reject a view. If it turns out that all the alternative views which avoid this access problem have worse flaws (as, e.g., formalist theories which have trouble capturing proof transcendent truth conditions and the role of math in the sciences plausibly do), this bullet might be worth biting. It might well be that some form of mathematical realism is worth adopting even if doing so requires us to accept a genuinely extra coincidence (though I think in such cases looking for some deeper explanation for an apparent coincidence is possible is still an interesting project which is motivated by general theoretical norms.

While formally a theory has an access problem to the extent no satisfactory explanation of the match between beliefs and belief-independent facts is possible, I will sometimes speak loosely and say that a theory faces an access problem when it *appears* that no such explanation is possible (though to be pedantic in only apparently faces an access problem). When it no longer appears that no such satisfactory explanation is possible I will say that the access problem has

been solved or dissolved. Thus, classical attempts to eliminate access worries like Modal-Structuralism, Quantifier Variance, Quineinism and Neo-Fregean view can be seen as attempts to solve (or partially solve) the access problem as conceptualized above¹⁶.

5.2 Philosophical Context

This way of understanding access worries can seem to require using an unacceptably imprecise notion of coincidence avoidance. However, I want to point out that the same imprecise notion plays an important role in scientific and philosophical reasoning.

We clearly have a practice of distinguishing certain parts of a theory as unattractive coincidences. And we take commitment to any such extra coincidences to be a (*ceterus paribus*) reason to disfavor a theory. Think of the kind of argument we might use to convince someone to stop believing in the Loch Ness monster. We generally wouldn't be able to derive the non-existence of the monster from beliefs we share with the Loch Ness conspiracy theorist, or locate a literal contradiction within their beliefs. Rather we would point out unattractive extra coincidences which the Loch Ness monster theory has to admit (the monster never shows up when someone has a really good camera, it only appears in pictures which could plausibly be faked etc.) which our Loch Ness monster hoax theory can't elegantly explain. We appeal to a kind of shared general epistemic norm, which says that one has *ceterus paribus* reason to avoid theories which posit certain kinds of (inexplicable) coincidences. What results isn't a deduction that the Loch Ness monster doesn't exist, but rather, *ceterus paribus* reasons for disfavoring its existence.

¹⁶Modal-Structuralism, Quantifier Variance, and Neo-Fregeanism help answer access worries (as characterized above) by suggesting that any logically coherent mathematical posits would express truths and thus explaining how any coherent mathematical beliefs we have correspond to mathematical truths (of course the issue of how we come to have coherent mathematical beliefs remains).

Admittedly, what makes something a coincidence is rather complicated. Coincidences aren't just facts posited by a theory which would otherwise be assigned low probability given the rest of a theory. For example, any particular long sequence of outcomes of a coin toss is unlikely, but we don't take total theories of the world which include the results of past coin tosses to be committed to an extra unattractive coincidence¹⁷. Nonetheless, spotting and rejecting such coincidences plays an important role in scientific and commonsense argument¹⁸, even when we can't appeal to anything like general Carnapian logic of induction. We might wish to have a tidy and uncontroversial criterion for when a theory counts as positing extra coincidences. However, we are all committed to using this kind of reasoning all the time, on a 'know it when you see it' basis. Thus, it seems reasonable to take these intuitions about theoretical badness at face value.

Finally, note that one can reasonably believe that coincidence-banishing explanation for some regularity is impossible for the realist without being able to explicitly deduce the impossibility from specific features of the realist's account¹⁹. So pressing an access worry does not require being able to cite particular features (causal inertness, lack of role in scientific explanations) which prevent the realist from providing an adequate coincidence-banishing explana-

¹⁷The feeling of coincidence/crying out for explanation seems related to an intuition that some other theory predicting the same things but with fewer dimensions of freedom should exist, but the question of a priori theory plausibility is an infamously hard one and I won't speculate about this more here.

¹⁸For example the clustering of the orbits of many trans-Neptunian objects has lead astronomers to hypothesize the existence of a 9th planet orbiting beyond 200 AU [19].

¹⁹To motivate this point, consider our intuitions about Tangram puzzles (which challenge us to re-arrange various sized triangles to form a particular shape). There are two quite different epistemic states one can be while judging that a particular Tangram puzzle seems impossible to solve. In some cases, one can point to specific features of a specified problem which allow one prove from attractive general principles that it is not solvable. In other cases one merely has the gestalt impression (arising from experience with related problems and general spatial intuitions) that a certain problem is impossible to solve. It's hard to deny that the latter intuitions, though not justifying full confidence, can rationalize assigning higher confidence to certain possibilities and acting accordingly. For example, analogous intuitions about what kind of proof strategies can and cannot be successfully used in a given case would seem to play an important role in rationally guiding mathematicians activities when they attempt to find a proof.

tion²⁰.

5.3 Contrast with other views

My proposal subsumes Field's account as the regularity R counts as an objectionable coincidences²¹. My proposal also subsumes Linnebo's account, at least if he is correct that R_{MS} is (despite the concerns I raised) entailed by a realist approach, as it too would count as an objectionable coincidence. On my account, if the realist cannot give any explanation (and hence cannot give any coincidence-banishing explanation) for R or (if Linnebo is correct) R_{MS} they face an access problem.

Stepping back, we can now see that there are two key novelties which distinguish my proposal from Field's and Linnebo's:

First, my version of the access problem identifies a holistic problem with the realist's account and thus (as noted above) can't be dismissed by explaining one type of accuracy in terms of another, equally problematic, type of accuracy. To solve my version of the access problem, one has to remove the appearance of commitment to positing *any* extra coincidences.

This holism allows my formulation of the access problem to avoid the trivializing explanations TRIV and EV-MOR above. We can allow that TRIV and EV-MOR do, in some sense, explain human possession of true beliefs (as witnessed by the fact that there are some contexts where analogous stories would provide a satisfying explanation) but we can account for the fact that these explanations don't seem to address access worries by noting that they seem

²⁰Thinking about access worries as being justified by a gestalt impression rather than a single salient feature of realist views is especially important insofar as previous discussion of access worries has often centered around a combination of features, e.g., non-analyticity and lack of scientific role which work together to jointly block various known strategies for explaining knowledge of a given domain from working in the mathematical/moral case. See Justin Clarke-Doane's [3].

²¹That is anything that has an access problem in Field's sense plausibly has an access problem in mine (but not vice versa).

to leave us with an extra coincidence, which other attractive but less realist theories regarding the same domain do not face. Specifically, TRIV explains our accuracy about realist mathematical facts by appealing to an unexplained coincidental-seeming match between the mathematical reasoning method Σ and realist mathematical facts. And EV-MOR explains our good intuitions about morality by appealing to an unexplained match between game theoretic optimality and objective moral facts.

Second, my version of the access problem involves a comparative element which Field's and Linnebo's approach don't. While a philosophical theory either does or doesn't allow for an explanation of R or R_{MS} (and thus does or doesn't face an access problem), on my approach one theory can be preferred to another as it requires accepting fewer coincidences. Because of this comparative element, we should *not* think of access worries as invoking an epistemic requirement to 'consign to the flames' every theory that posits a coincidence (analogous to Hume's famous empiricist exhortation to reject all concepts that weren't suitably related to experience[8]). Instead, access worriers appeal to general norms in favor of *reducing* the number of coincidences one is committed to positing insofar as this is compatible with other epistemic goals. For example, even if our knowledge of inductive generalization raises an access problem on its own right (maybe even an insoluble access problem) we can still invoke inductive generalization to dispel our access worries regarding a domain like mathematics (as no rival theory would dispel the coincidence that the future seems to behave like the past). Thus, in contrast to earlier accounts, my view allows theories to suffer access worries to varying degrees depending on the number and implausibility of the coincidences the theory posits.

6 Answering Clarke-Doane’s Challenge

With this explication of the access problem on the table, let us now turn to Justin Clarke-Doane’s argument in ‘What Is the Benacerraf Problem?’[4] that there can’t be an access problem for mathematics, morals (and other domains with similar features). As Clarke Doane puts it in [4]

“The difficulties surrounding the Benacerraf Problem are actually very general. They infect formulations of it aimed at moral realism, modal realism, logical realism, and philosophical realism. But they infect much more than this. They infect any argument which grants the (actual) truth and (defeasible) justification of our beliefs from an area and seeks to undermine those beliefs, so long as the area F meets two conditions. Those conditions are:

1. The F-truths would be metaphysically necessary.
2. There is a plausible explanation of our having the F-beliefs that we have which shows that we could not have easily had different ones.”

I take Clarke-Doane’s argument for this thesis to have essentially the following structure.

1. If there were a genuine access worry for realists (about some domain), it would undermine our justification for holding all beliefs of a certain kind K ²².

²²I take Clarke-Doane’s ‘beliefs of a kind K ’ relevant to access worries for some form of realism about some practice P (e.g., of talking about mathematics or morals) to be the beliefs which *these realists* take to be expressed by ordinary utterances within that practice. With some admitted exceptions like Hartry Field’s nominalism, philosophers pressing an access worry generally take access worries to undermine (only) our *realist interpretation* of assertions about some domain and demand we substitute some alternate interpretation. Thus, what access worries undermine is our justification for accepting certain metaphysical propositions which realists (but not anti-realists) take to be expressed by ordinary moral/mathematical assertions.

2. “Modal Security: If information, E , undermines all of our beliefs of a kind, K , then it does so by giving us reason to doubt that our K -beliefs are both sensitive and safe.”[4]²³.
3. Mathematical and moral realists can give a trivial explanation (from broader facts they believe) for both the safety and sensitivity of the relevant K beliefs.

Justin Clarke-Doane supports this claim by describing a pair of strategies for providing such explanations. I won't say more about his strategy for explaining sensitivity because I grant that it works. But I will discuss (and raise some problems for) his strategy for explaining safety below.

4. If realists can explain the safety and sensitivity of our K beliefs (from broader facts the realist believes), then access worries cannot give us reason to doubt the safety and sensitivity of these beliefs.

Thus, Clarke-Doane concludes, there can be no legitimate access worry for mathematical or moral realism. However, I will argue that arguments above give us strong reason for denying 4. There are also some reasons to doubt the safety claim in point 3. Note that Clarke-Doane's argument concludes that it would be in principal impossible for any philosophy of mathematics to face an access problem, even if it is impossible to banish the appearance of coincidence and is thus in conflict with my view even if you think that the access problem for the right philosophy of mathematics is ultimately tractable.

²³Here, as usual, **sensitivity** means that if the content of some of our K beliefs had been false we wouldn't have still had these beliefs. In possible worlds terms: for every K belief, the closest possible worlds where that belief is false is one in which (our counterparts) don't hold that belief. And the **safety** requirement is also cashed out in a largely familiar way, as a claim that we couldn't have easily formed false K beliefs, or (sometimes) that we couldn't have easily formed false K beliefs “by using the [same] method we used to form our actual K [beliefs]” In possible world terms, all worlds in a certain sphere of close possible worlds where we continue to use the same K -belief forming ‘method’ are ones where we continue to have true K beliefs.

6.1 Not All Mathematical Realists can Trivially Explain Safety

Not all mathematical realists can give the trivial explanation for safety that Clarke-Doane proposes. Thus (even if the rest of Clarke-Doane’s argument worked) it would be overclaiming to say that it shows there is no access problem for mathematical realists in general.

Specifically Clarke-Doane argues that mathematical realists have no problem explaining safety because they can say something like the following (along the lines of TRIV above). We could not have easily been wrong about mathematics, because mathematical truths are all necessary and we could not have easily had ‘different’ mathematical beliefs. Thus, all close possible worlds where we have the same mathematical beliefs will also qualify as ones where we continue to have true mathematical beliefs²⁴.

As stated, I think this argument needs a little more careful explication. There is obviously a sense in which we *could* have easily had different mathematical beliefs. Clearly if different mathematicians had died in duels or different problems had attracted their interest, this could have easily lead to different areas of mathematics being researched and different approaches being tried. So surely the collection of theorems provable in ZFC which mathematicians accept *could* easily have been different. But presumably that is not the kind of minor

²⁴Clarke Doane writes, “For typical contingent truths K , our K -beliefs can fail to be safe in two ways (assuming their actual truth). They can fail to be safe, first, if it could have easily happened that the K -truths were different while our K -beliefs failed to be correspondingly different. They can also fail to be safe, second, if it could have easily happened that our K -beliefs were different while the K -truths failed to be correspondingly different.

But if the mathematical truths are the same in all nearby worlds, and we are granted the (actual) truth of our mathematical beliefs, then our mathematical beliefs may well be safe even if standard mathematical realism is true. Again, if the mathematical truths could not have easily been different then our mathematical beliefs cannot fail to be safe in the first way (assuming their actual truth). Moreover, if we could not have easily had different mathematical beliefs (regardless of whether such beliefs would have been true), then they cannot fail to be safe in the second way (again, assuming their actual truth). But there are reasons to think that we could not have easily had different mathematical beliefs. ”

difference which Clarke-Doane has in mind.

We can more charitably interpret him as saying that there is some property which our current mathematical beliefs have (say, being derivable from ZFC) such that both: all beliefs with this property are necessarily true and we could not have easily come to form beliefs without this property.

However, even once we have made this distinction, it remains overclaiming to say that Clarke-Doane's sensitivity explanation can be used to defend mathematical realism in general. The iterative hierarchy conception of sets was developed by looking for *some* coherent structure which lets one avoid Russell's paradox, not by considering all choices of fundamentalia that would do the requisite work and picking the best one. This suggests that, had history gone differently, we might well have accepted different mathematical foundations, e.g., category theory or perhaps even something like Quine's New Foundations [16] instead of ZFC. However, certain straightforward forms of Platonism hold that a different choice in mathematical foundations would have made our claims factually inaccurate²⁵. Thus, it appears that, according to certain forms of realism, we *could* have easily had false mathematical beliefs.

Some brief remarks Clarke-Doane makes later in the paper suggest a way of complicating the simple argument I attribute to him above. He suggests that our mathematical beliefs may be abductively tied to proto-mathematical beliefs which are fixed by evolution and innate. Specifically, he writes: "Our 'core' mathematical beliefs might be thought to be evolutionarily inevitable. Given that our mathematical theories best systematize those beliefs, there is a 'bootstrapping' argument for the safety of our belief in those theories. Our 'core' mathematical beliefs are safe; our mathematical theories 'abductively follow' from those; our abductive practices are 'safe' ... so, our belief in our mathemat-

²⁵Some interpretations of realism, like plenitudinous platonism, quantifier variance and modal structuralism, regard different choices in fundamentalia as equally good, but Clarke-Doane isn't only committed to defending them.

ical theories is safe.” [4] Does this mean that, as Clarke Doane argues, even on straightforward forms of platonism we couldn’t have easily been wrong?

However, considering alternate ways that early 20th century mathematics could apparently have gone (as above) strongly suggests that, to whatever extent our mathematical practices are evolutionarily inevitable, it is not enough to determine the particular choice of fundamentalia (e.g., to force the choice of ZFC rather than category theory, as in the case above). Yet, straightforward versions of mathematical realism hold that the truth of our mathematical beliefs depends on this choice. Thus, I think Clarke-Doane’s trivial explanation of safety can’t be applied to the defense of some forms of mathematical realism.

6.2 Explanations From Realist Assumptions Don’t Assuage Access Worries

Now let’s turn to my other point of disagreement with Clarke-Doane: his claim (premise 4 above) that access worries cannot call into doubt the safety or sensitivity of the realist’s beliefs, if the realist can explain the safety and sensitivity of her beliefs from other premises she accepts.

Of course, an explanation of safety from *generally accepted premises* would block any access worries, as safety implies truth. But, as access worries undermine our belief in the premises the realist uses to justify safety, Clarke-Doane’s trivial explanation of safety doesn’t qualify as an explanation of this kind.

As we saw above, one can raise doubts about a realist theory by pointing out implausible coincidences lurking in it. And this seems to be possible even when the theory allows for an explanation safety and sensitivity (contra premise 4 above). So I’ve suggested that we must distinguish the existence of an internally coherent deduction of safety/security from realist assumptions and lack of an internal tension in the realist’s story.

This point can be motivated by considering realist theories which invoke explanations like EV-MOR and TRIV. Proponents of such theories can explain safety and sensitivity given the truth of other premises which they accept. However, the strong intuition that there is something like an access worry or internal explanatory problem remains. After all, it seems awfully convenient for the realist that the evolutionarily preferred ratio of family favoritism is also the morally correct ratio (as postulated in EV-MOR).

However merely considering such apparent counterexamples to premise four surely wouldn't move Clarke-Doane. He discusses explanations like EV-MOR, and is well aware that his position (that such trivial-seeming explanations can block access worries) looks somewhat counterintuitive. He even notes that the assumptions about the evolutionary robustness of moral beliefs he uses to trivially explain safety (from a moral realist point of view) and thus *block* access worries are the same ones which Sharon Street uses to *press* access worries for moral realism[4].

This leaves us a *prima facie* puzzle of why Clarke-Doane accepts claim 4. I take Justin Clarke-Doane to resist the intuitive pull of such worries on the grounds that various attempts at explaining *how* access worries give us reason to doubt realism collapse²⁶. Specifically, they either face the Scylla of merely demanding an explanation for accuracy/safety/security from some premises which the realist believes (and hence allowing for responses like those above), or the Charybdis of making justificatory demands so strong that applying them consistently would equally appear to rule out many kinds of uncontroversial knowledge, e.g., knowledge that the sun will rise tomorrow.

I take myself to be answering this challenge head on, by developing a clear

²⁶Although Clarke-Doane represents access worries as presenting new evidence, I think they are – like mathematical arguments – making an a priori philosophical point (hence presenting facts which they think an ideal Bayesian agent would already have recognized rather than presenting new evidence on which such an agent would update). This distinction may, in part, explain the different conclusions we reach about explanations like TRIV and EV-MOR.

third option: a different account of what access worries are and how they undercut realism. My coincidence-avoiding story fits access worries into a larger framework of general norms for a priori theory choice (favoring coincidence avoidance). By formulating the access worry as a demand for internal coherence, I avoid identifying access worries with unacceptably strong demands for explanations of safety and reliability from indubitable premises. But, by cashing out this apparent failure of internal coherence as a tension within the realists' overall web of beliefs (including their norms of coincidence avoidance), rather than an inability to internally explain *any one thing*, we also avoid letting trivializing explanations (like the ones Clarke-Doane proposes) block access worries.

Alternately one can read Clarke-Doane as arguing (from philosophers' failure to 'cash out' the coincidence avoidance intuitions behind access worries in other, less controversial, terms) that our coincidence avoidance intuitions become unreliable when regularities 'suitably involving' necessary truths are at issue. Note that our reliability about mathematics (or morals) is itself not a necessary truth, but rather a match between contingent truths and necessary truths that seems to cry out for explanation, much as it would if an apparently fair series of coinflips spelled out the binary expansion of π to a thousand places. So even if our intuitions about coincidences about necessary truths are unreliable it's unclear they would invalidate our intuitions about the coincidences involved in the access problem.

However, Clarke-Doane doesn't present much reason for thinking that analyzing the notion of coincidence avoidance in cases where both sides of the relevant coincidence are contingent truths is much easier. Attempts to analyze paradigmatic contingent cases of crying out for explanation in terms of equally questionable notions like that of extraordinary types[17]. In contrast, there are plenty of good paradigms for how to think about coincidence avoidance that

would allow it to apply to both necessary and contingent regularities, e.g., in terms of preferring theories that have fewer degrees of freedom and a general scientific desideratum to prefer theories that unify [10].

Much more damningly, our intuitions about coincidence avoidance become incoherent when applied to necessary truths seems to conflict with existing mathematical methodology, which does seem to use explanation-seeking and coincidence avoidance to guide research perhaps (more controversially) our choice of new axioms [1][11]. For example, the history of John Conway’s ‘Monsterous Moonshine’ conjecture is provides a particularly dramatic illustration of how noting a striking relationship between two pure mathematical facts and expecting a deeper unified explanation can lead to important discoveries *even when a proof of both facts already exists*.

Similarly, in philosophy we seem happy to accept that avoiding coincidence in the sense of favoring theories that unify many explanations with few resources²⁷. Clarke-Doane hasn’t shown that there’s any *principled and theoretically attractive line* which carves off the specific intuitions about coincidence avoidance and necessary truths which he wants us to be suspicious of (those driving access worries) from general methods of reasoning which are attractive and ubiquitous in philosophy and mathematics. Therefore, absent a stronger argument that such reasoning leads us astray, I don’t see any reason to eschew its use.

²⁷We think it’s a good thing when a theory of justice or supererogation unifies and explains a lot of things using only a few resources, and this concern with providing elegant explanation seems continuous with avoiding apparent coincidences. And we think it’s a puzzle that psychophysical laws and biology line up with (apparent) facts about what responses to pleasure/pain qualia are rational/appropriate, so that the physical basis of pleasant qualia cause seeking behavior and pain qualia cause avoidance behavior, rather than vice versa – and a real advantage of attitudinal hedonic theories (on which painful experiences are just ones that are avoided for their own sake) that they can offer an explanation for this.

7 Looking forward: Are Access Worries Tractable?

Let me end this paper by briefly discussing my coincidence-avoidance approach suggests that philosophers could settle disputes about apparent access problems.

My proposed formulation of the access worry is, in a sense, less definite than previous formulations, so one might fear that renders debates about access worries less philosophically tractable. The existing (more specific) formulations of the access worry tend to derive to access problems from powerful general philosophical claims and/or argue that certain specific features of the realist picture make knowledge impossible. This provides a natural line of attack for realists attempting to answer access worries: they can dispute the relevant general claims. In contrast, I have suggested that we should think about access worries by invoking a shared implicit grasp of what qualifies as an undue coincidence together with an overall impression of the possibility for giving explanation. So adopting my proposal might seem to motivate pessimism about the prospects of resolving access worries.

In itself, this unfortunate consequence would not be a reason to think my story is wrong, but only a reason to be unhappy. Luckily, such pessimism is not warranted, as there are alternative means of removing the appearance of unattractive coincidence²⁸. For instance, consider the very rough story provided by Quinean empiricism about mathematics²⁹. On the Quinean picture, we acquire largely true beliefs about mathematics in the course of acquiring accurate scientific beliefs as a whole – by adopting empirically motivated scientific

²⁸While there are no obvious examples of such coincidence banishing stories for plausible access worries it is easy to find them in other areas, e.g., if someone objected to the claim that the Loch Ness monster couldn't possibly exist because it would entail the unattractive coincidence that all the separate claimed sightings described similar creatures one could respond by pointing out that sightings of the modern Loch Ness monster all occurred after media reports and the supposed witnesses based their descriptions on these accounts.

²⁹Here I will be discussing Quinean Empiricism where this is understood as a story about the actual history of how human beings got accurate beliefs about mathematics, rather than (as I take Quine to have intended it to be) a story about how our beliefs are justified.

theories which force us to quantify over suitable mathematical objects. There are well known problems for this Quinean story³⁰ but it illustrates the kind of approach that could alleviate access worries in a way other than attacking some general claim used to derive the access worry.

In addition, we should also note historical failure to discover any plausible examples of a certain kind of explanation can itself gradually increase access worries on my account³¹. Thus, while my account doesn't leave us bereft of any way to repudiate access worries, it also doesn't obviously make such worries any less forceful. Indeed it provides a way for access worries to get worse as well as better.

8 Conclusion

In this paper I have proposed way of understanding intuitive access worries, which lets us avoid the problems of trivializing answers and over-demanding philosophical presuppositions which beset currently popular approaches formulations of access worries. Specifically, I have suggested that we should think of access worries as invoking a general, shared, norm of a priori theory choice, which counsels us to avoid positing inexplicable coincidences (*ceterus paribus*). This formulation relates access worries to more general norms of 'coincidence reduction' and scientific theory choice.

While this approach reflects familiar intuitions about the access problem, my analysis makes important advances in fending off quietist arguments against the existence of a coherently formulable, non-skeptical access worry.

³⁰For example, we seem to get mathematics right in advance of scientific applications, and we seem to know about the existence of mathematical objects in areas such as higher set theory which may forever lack relevant scientific applications).

³¹I say this with the caveat that philosophical progress is often very slow, so (as noted in the previous section) it might take a long history of many failed attempts to create any significant confidence that this challenge cannot be answered.

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