Baker's Rocketship And The Mathematical Nominalist's Real Problem With Physical Magnitudes

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Abstract

Nominalists face the challenge of paraphrasing scientific theories involving physical magnitudes without quantifying over mathematical objects. Measurement-theoretic uniqueness theorems offer a promising path—but nominalist strategies that rely on them face three major difficulties: (1) they may fail to capture the explanatory power of mathematical formulations; (2) they break down when magnitudes are sparsely instantiated; and (3) they only fix quantities up to scale, not absolute values. This paper develops a modal if-thenist paraphrase strategy that addresses all three challenges. The resulting approach not only avoids classic Quinean indispensability worries but promises to match — or even improve on — the explanatory virtues of Platonist accounts in certain cases.

1 Introduction

Indispensability arguments for mathematical Platonism maintain that (in one way or another) we cannot adequately make sense of our current scientific knowledge without accepting the existence of mathematical objects. The classic (Quinean) indispensability argument holds that we need to quantify over mathematical objects to *literally state* our best scientific theories, and this commits us to the existence of such objects. And explanatory indispensability arguments[1] point out that mathematical facts do the heavy lifting in certain scientific explanations, and maintain that mathematical objects are needed to *best explain* certain scientific facts.

A key source of worry about nominalists' ability to answer classic indispensability worries [23, 10, 16, 8] directly (i.e., by providing a paraphrase) concerns physical magnitude statements (i.e., statements about lengths, temperatures and the like). When formalizing a theory like Newton's law of gravity, the Platonist can appeal to a length function, which pairs each spatial path with its length-in-meters (a certain real number). And a nominalist paraphrase must simulate or replace such talk of a length function where it appears.

An influential and scientifically motivated tool for approaching this challenge has been appeal to measurement theoretic uniqueness theorems. Such theorems show that, given sufficiently rich physical structure, we can uniquely characterize physical magnitude functions (up to choice of scale) by requiring they respect certain observable relations. For length, if we can assume spatial paths exhibit sufficient variety, we can pin down the length function by requiring it respect relations like 'path p_1 is at least as long as path p_2' and 'the combined length of paths p_1 and p_2 equals the length of path p_3'. Combining appeal to such notions with classic modal if-thenist nominalization strategies promises to offer a principled, empirically grounded way to understand how physical magnitude claims get their content.

However, three significant challenges arise. First, there's an explanatory challenge: even adequate nominalistic paraphrases may be explanatorily inferior to their Platonist counterparts. Second, there's a sparse magnitudes problem: many physical magnitudes don't appear instantiated in sufficiently varied ways for measurement-theoretic uniqueness theorems to apply. Third there's a unit-fixing problem highlighted by Baker's Rocketship argument: measurement-theoretic approaches only pin down magnitude functions up to scaling transformations, yet adequate paraphrases of deterministic theories require fixing absolute scales, not just ratios.

In this paper, I will attempt to provide a solution to all three challenges.

In §2 I'll review basic indispensability worries and introduce the relevant modal notion (a kind of logical possibility). In §3 I'll formulate a basic modal if-thenist nominalization strategy[1] and argue that (where this paraphrase strategy can be applied) it promises to address explanatory worries by providing logical regimentations which are explanatorily at least as good -indeed arguably better- than classic platonist paraphrases. In §4 I'll review how physical magnitude statements pose a continuing problem for applying this theory (and nominalist paraphrase in general). In §5 I'll argue that we can solve this problem by employing formal 'cheap tricks'[2] .

In §6 I'll review Baker's recent rocketship argument against comparativism and argue that it highlights problem for the nominalist paraphrase strategy outlined so far – about whether it can adequately pin down the exact intended behavior of a physical magnitude function, rather than merely describing it up to multiplication by a constant. I'll then suggest a way of solving this problem by making a small modification[3].

2 Background and Motivation

2.1 Modal If-thenism

What does it mean to succeed in nominalistically paraphrasing our best physical theories in response to Quinean and explanatory indispensability challenges?

[1] c.f. [16, 22]

[2] This element of my proposal attempts to better motivate and develop ideas sketched in [5].

[3] Admittedly, the paraphrases produced by this strategy won't be helpful to every nominalist. For example, philosophers who reject mathematical nominalism as part of a general physicalist project will probably reject my key notion of logical possibility as insufficiently physical. However, as Putnam notes[22], in many contexts we can seemingly equally well take either a modal or a platonistic perspective on pure mathematics. And certain puzzles (concerning the Burali-Forti paradox and the height of the hierarchy of sets) appear to favor a modal approach to pure higher set theory [16, 19]. So one might be interested in whether we can take a similarly modal perspective on mathematics as a whole.

Much could be said on this topic. But, for the purposes of this paper, I'll consider whether we can provide a nominalistic paraphrase strategy T which produces a translation $T(\phi)$ of our total best physical theory ϕ, with the following pair of good features.

- **Correct possible-worlds truth-conditions according to the Platonist:** T transforms every Platonist sentence ϕ (which it applies to) into a nominalistically acceptable sentence $T(\phi)$ which Platonists must regard as capturing the non-mathematical content of ϕ (by being true at exactly the same metaphysically possible worlds as ϕ).

- **Equal intuitive physical explanatory power :** T preserves intuitive explanatory power (as regards physics, if not metaphysics). $T(\phi)$ is intuitively able to explain all the same physical facts that ϕ does.

How might one try to provide such a paraphrase? An if-thenist approach to nominalistically paraphrasing mathematical claims has enjoyed enduring popularity, going back through Hellman, Putnam, Russell and perhaps even (on a certain interpretation) Aristotle.

And modal forms of if-thenism, as developed by Putnam and Hellman[17, 22] deploy the following basic ideas.

- First, we come up with a sentence D which completely pins down the mathematical and quasi-mathematical structures the Platonist (but not the nominalist) believes in. For example, in the case of the natural numbers, this sentence might be a conjunction of the second-order Peano Axioms characterizing the natural number structure (written using the conditional possibility operator[4]).

[4] See [5] for a demonstration of how to replace second-order quantification with the conditional logical possibility operator.

- Then we nominalistically formalize each apparently platonistic scientific theory ϕ (e.g., Newton's law of gravitation) as saying something like this. (It's in some sense necessary that) **if** there were objects satisfying D then ϕ would be true.[5][6]

So, basically, the modal if-thenist strategy is to have $T(\phi)$ assert that ϕ would be true if we supplemented the non-mathematical world with the mathematical (and applied mathematical) objects whose existence the Platonist assumes.

What modal notion shall we use for fleshing out this approach? One popular choice is to invoke a notion of logical possibility (approximately interdefinable with entailment) as a primative modal notion. Hellman developed an influential version of Putnamian if-thenism in [16] along these lines. For pure mathematical purposes, we can easily think of the modality invoked by Hellman's paraphrases invoking as logical possibility.

However, as Hellman is well aware, appealing solely to logical possibility creates difficulties with paraphrasing applied mathematical statements. We can't just talk about what's logically possible or necessary full stop if we want to paraphrase contingent claims whose truth-value depends on the state of the actual world like 'there are a prime number of bugs on a rug'.

To address this problem, Hellman tries to appeal to metaphysical possibility and counterfactuals where mathematical objects exist. But this strategy has drawn criticism from figures like Linnebo[20]. Is it really clear that (as Hellman's strategy needs to assume), counterfactual scenarios where mathematical objects/structures *do* exist are ones where all relevant non-mathematical facts

[5] In cases where we have a categorical description of the relevant structure (i.e., any two structures satisfying the description would have to be isomorphic to each other), this gives bivalent truth conditions for all pure mathematical statements. Note that when it's necessary to use second-order quantification to pin down a categorical conception of the relevant structure, we can do this purely in the language of conditional logical possibility as shown in [5].

[6] We may also want our nominalist paraphrase to include the claim that it is (in the relevant sense) possible for the relevant Platonist claim to be true[16, 5]. In this case, our paraphrases will take the following form $T(\phi) : \Box(D \to \phi) \wedge \Diamond D$.

about the actual world we are trying to describe (about bugs on rugs etc) would remain the same?[7]

One way to avoid this problem of unwelcome commitment to controversial counterfactuals is to use a different (independently useful and motivated) modal notion to write such paraphrases: the *conditional logical possibility* operator.

2.2 Conditional Logical Possibility

In this section I will briefly explain what the conditional logical possibility operator is, and why I think it is a promising candidate for use when formulating modal if-thenist paraphrases in response to Quinean and explanatory indispensability challenges.

To motivate this notion, first recall that there's independent reason for accepting a primitive notion of *logical possibility simpliciter* ($\Diamond$) interdefinable with logical entailment [8]. When evaluating logical possibility in this sense, we ask what is possible while ignoring all constraints on the total size of the domain, and consider all possible ways of some choosing n-tuples from this domain as extensions for atomic relations.

Now consider a situation where there are three cats and two blankets. Could it be that each cat is sleeping on a different blanket? No, as per the pigeonhole principle. In this situation, there's some appeal to saying that it's logically

[7] Worries have also been raised about whether it would be metaphysically possible for there to be either mathematical objects or non-mathematical objects plentiful enough to form structures satisfying the axioms of set theory etc. For example if there are metaphysically necessary limits on the cardinality of the dimensions of space, it may be that not be metaphysically possible for there to be enough objects to satisfy any adequate description D of the intended height of the hierarchy of sets[21]

[8] I follow [11] in taking the $\Diamond$ of logical possibility as a primitive modal notion (that's a logical operator).

Admittedly there's a fruitful tradition of identifying logical possibility with having a set theoretic model for various mathematical purposes (and validity with not having a countermodel). However, there are independent reasons[14, 15, 6, 9, 13] for thinking we have prior grasp on the notion of logical possibility.

Also, one might feel (with Boolos) that, "one really should not lose the sense that it is somewhat peculiar that if G is a logical truth, then the statement that G is a logical truth does not count as a logical truth, but only as a set-theoretical truth"[6].

impossible that each cat is sleeping on a different blanket. But, of course, it's not logically impossible simpliciter for each cat to have its own blanket. A scenario in which four cats are sleeping on four blankets is logically coherent. Rather, we might say that it's logically impossible for each cat to have its own blanket *given the structural facts about how cathood and blankethood apply.* So we seem to have a notion of logical possibility which doesn't just depend on general facts about logical combinatorics but also on preserving structural features of how certain relations (e.g., cathood and blankethood) apply within a given domain.

Since this notion is distinct from plain logical possibility, we might call it conditional or structure preserving logical possibility. Recent philosophical work has used a conditional logical possibility operator $\Diamond_{R_1,\ldots,R_n}$ to do a few different jobs. For example, [3] uses it to reconceptualize the kind of knowledge of logical coherence needed for choosing acceptable mathematical posits, for the purposes of answering access worries. And [5] advocates reformulating potentialist set theory using this notion, and proposes powerful axioms for reasoning about conditional logical possibility capable of reconstructing (resulting potentailist translations of) standard ZFC set theory.

When evaluating conditional logical possibility ($\Diamond_{R_1 \ldots R_n}$) we consider arbitrary domain sizes and ways for most relations to apply, as when evaluating logical possibility simpliciter. However we hold fixed (structural facts about) how the subscripted relations $R_1 \ldots R_n$ apply.

So, for example, we can express the above claim about cats and blankets as follows.

$\neg\Diamond_{cat,basket}$ [Each cat is sleeping in a basket and no two cats are sleeping in the same basket.]

This says that it's logically impossible (holding fixed structural facts about how cathood and baskethood actually apply) that each cat is sleeping in a different basket as follows.

We can also nest conditional logical possibility operators, and talk about the conditional logical possibility of claims which are themselves described in terms of conditional logical possibility as discussed in [4, 5][9]

2.3 Motivating Case: Three Colorability

Using this $\Diamond_{R_1 \ldots R_n}$ operator in our modal if-thenist paraphrases eliminates the Hellman's problem about dependence on controversial counterfactuals or metaphysical possibility claims. For, we no longer need to worry about whether the scenarios we need to consider where our description D of supposed platonistic structure is satisfied, are truly metaphysically possible. We also don't need to worry about whether these scenarios preserve actual world facts about nominalistic relations (i.e. relations whose extension the platonist and nominalist agree on) apply, since we can insure this by suscripting the relevant relations [10].

It also promises to help us give logical regimentations for certain kinds of mathematical explanations of physical facts (approximately those Lange calls distinctively mathematical 'explanations by constraint'[18]). To see what I mean, consider the following example of a distinctively mathematical expla-

[9]

For example, it's logically possible that there are three cats and two baskets. So, it's logically possible for 'cat' and 'basket' to apply in a way that makes it logically impossible (given the structural facts about how cathood and baskethood apply) that each cat is sleeping in a different basket. And we can write that claim as follows.

$\Diamond \neg \Diamond_{cat,basket}$ [Each cat is sleeping in a basket and no two cats are sleeping in the same basket.]

Note that here the interior expression $\Diamond_{cat,basket}$ makes a claim about the structure of the cats and baskets in whatever possible scenario is being considered. It doesn't preserve the way these terms apply in the actual world. See appendix A and [4, 5] for a more technical details.

[10]So, for example, we no longer need to worry that our if-thenist paraphrases of applied mathematics only work if we assume the metaphysical possibility of worlds with enough objects to form intended models of claims in higher set theory, or that all/all the closest such metaphysically possible worlds would be ones where facts about the cats and blankets we want to describe have not changed.

nation of a physical fact – which doesn't just explain the explanandum, but also (in a sense) shows it to be necessary in a way that transcends the necessity of mere physical laws).

Imagine a case where a certain physical map (perhaps one with infinitely many countries) has never actually been three-colored. A good explanation for this fact might be that (in a mathematical sense) the map isn't three colorable. A Platonist might express this idea as follows.

> **platonistic Non-Three-Colorability**: There is no function (in the sense of a set of ordered pairs) which takes countries on the map to numbers $\{1, 2, 3\}$ in a such a way that adjacent countries are always taken to distinct numbers.

However, we now have an additional nominalist version of the above three-colorability explanation to consider.

> **Modal Non-Three-Colorability**: $\neg\Diamond_{adjacent,country}$ Each country is either yellow, green or blue and no two adjacent countries are the same color.

And the above modal explanation arguably at least as good, or indeed better than the original Platonist explanation [11]. In particular, one might argue that the platonistic non-three-colorability claim is defective (or at least unable to stand on its own) in the following sense. One can only intuitively explain the fact that a physical map is not three-colored because we assume a certain relationship

[11] What about capturing relationships between pure mathematics and statements about concrete physical systems (e.g., how reasoning about the natural numbers can imply and explain facts about all graphs and thereby the fact that if there are more than two islands with even numbers of bridges Köeningsburg then it is not possible to talk a walk which crosses each Köeningsburg bridge exactly once)? The simple nominalization example above does not make it obvious how we can capture all such reasoning, but the more general uniform nominalist paraphrase strategy proposed below will allow this, as discussed in [**?**]

between set existence facts and the modal facts referenced above. Specifically, we assume that there are functions corresponding to *all possible ways* of pairing countries with one of the numbers 1, 2 or 3, and hence all possible ways of choosing how to color these countries. If we suspend judgment on this claim, inference from the non-existence of a certain kind of *set* to the claim that the map isn't actually three-colored begins to look unjustified.

Thus, one might argue that the real explanatory work here is being done by the modal principle; claims about what mathematical objects (e.g., sets coding three coloring functions) exist don't seem to add explanatory value[12]. One might also claim it as an advantage that the modal nature of the nominalist paraphrase matches ordinary language better than platonistic paraphrases do. We tend to express thoughts like the non-three-coloring explanation above modally, by talking about maps being three color*able*, rather than ontologically, by talking about maps *having three colorings*.

In any case, I hope considering the above toy explanation provides a motivating example of how logical tools used in potentialist set theory (the conditional logical possibility operator) can help us nominalize Platonist scientific theories in a way that preserves (or improves) their explanatory and unifying power.

3 Nominalist Paraphrase

Now let's turn to the task of providing a general nominalistic paraphrase strategy — which preserves explanatory and unificatory virtues as above. Recall that we want to produce a nominalist theory $T(\phi)$ which the Platonist must regard as true at the same possible worlds as a face value (Platonist) version of our best total scientific theory ϕ.

To create such a paraphrase using the if-thenist approach sketched above,

[12]See [5] for more argument on this point.

we need a sentence D, which precisely specifies the intended behavior of all the extra structure the Platonist believes in, given the nominalistically acceptable facts. Specifically, D should have the following properties.

- The Platonist takes D to be a metaphysically necessary truth,
- D uniquely pins down how all the platonistic relations (i.e., relations whose extensions the Platonist and nominalist disagree on) are supposed to apply at each metaphysically possible world – given the facts about how some finite list of nominalistic relations $N_1 \dots N_m$ apply at that world[13].

I will call such a sentence D a definable supervenience sentence. And I will say that the application of some platonistic vocabulary $\vec{P}$ definably supervenes on the application of some nominalistic vocabulary $\vec{N}$ (i.e., relations whose extensions the Platonist and nominalist agree on) when we can write such a sentence D.

So, for example, a definable supervenience sentence for the platonistic vocabulary in a scientific theory involving natural numbers will include a categorical description of the natural numbers. And if we want to translate Platonist claims involving quantification over a (supposed) layer of sets whose elements are goats, this definable supervenience claim will typically involve statements that imply that sets (of goats) are extensional and some version of the idea that there's a set of goats corresponding to 'all possible ways of choosing' some of the goats[14].

If we can find such a definable supervenience sentence D, we can nominalistically translate every sentence ϕ which only employs relations in $\vec{P}, \vec{N}$ (and has all quantifiers restricted to objects that appear in the extension of at least one of the relations in $\vec{P}, \vec{N}$[15]). For the truth value of all such sentences ϕ will

[13] That is, it should not be logically possible for two scenarios to both agree on structural facts about how nominalistically acceptable relations $N_1 \dots N_m$ apply (i.e., have isomorphic $N_1 \dots N_m$ structures) and satisfy D, while disagreeing about the platonistic structures.

[14] This concept turns out to be easy to express in the language of conditional logical possibility, as discussed in [5]

[15] More formally, those objects which take part in some tuple satisfying one of these relations.

be completely determined by the structure of objects satisfying the platonistic and nominalistic relations $\vec{P}, \vec{N}$. And our definable supervenience description D precisely pins down all the extra structure of pure and impure mathematical objects and relation the Platonist believes in at each metaphysically possible world (typically by specifying both axioms directly characterizing pure mathematical structures and intended relationships between the desired extensions of platonistic and nominalistic vocabulary)[16].

Accordingly, we can nominalistically paraphrase any sentence ϕ (for which we can write a suitable definable supervenience condition D) as follows.

$T(\phi) = \Box_{\vec{N}}(D \rightarrow \phi)$

Intuitively, this says that it's logically necessary, given the structure of objects satisfying the nominalistic relations $\vec{N}$, that *if* there were (objects with the intended structure of the) relevant supposed mathematical objects, then ϕ would be true[17]

So, for example, consider the statement

GOATS 'There are some goats who admire only each other[18].

This sentence only appeals to sets whose elements are physical objects. One can uniquely pin down the intended structure of such sets (how 'set' and 'element' would apply within this structure), given the facts about how predicates picking out all the intended ur-elements apply (via the strategy for simulating second order quantification in [4, 5]). Thus one can write down a suitable definable supervenience sentence D that exactly pins down all platonistic structure relevant to this sentence. Thus we can apply our nominalistic paraphrase

[16]Note that, as one can categorically describe many core mathematical structures which cannot be categorically described using first order logic alone (like the intended natural number structure, via the conditional logical possibility operator, as discussed in [4, 5].

[17]Note that the Platonist must believe it is always logically possible to supplement the non-mathematical objects at each possible world with additional objects so that D is satisfied, for the Platonist thinks that D is a metaphysically necessary truth.

[18]Here I mean the version of this which a Platonist might express by saying: there's a collection/set of goats which only admire other goats in that collection.

strategy to get a sentence T(GOATS) with the following form.

$\Box_{goat,admire}$[There are (objects with the intended structure of) the sets of goats $\rightarrow$ There is a set of goats x, such that the goats in x admire only each other.]

This nominalistic paraphrase strategy satisfies the conditions for successful paraphrase set out in §3.

It satisfies **correct possible-worlds truth-conditions according to the Platonist**. For, from a nominalist point of view, $T(\phi)$ captures all the non-mathematical content that the Platonist *intended* to express by ϕ. Where it is defined, $T(\phi)$ is true at exactly those metaphysically possible worlds where the Platonist thinks ϕ is true.

And it (promises to) satisfy the equal intuitive physical explanatory power requirement extremely well. For (as discussed above §2.3) regimentations which appeal to logical necessity arguably do a better job than classic Platonist explanations in capturing the intuitive modal force of distinctively mathematical explanations for physical facts. And because the above proposal allows a uniform paraphrase strategy (if we find a single definable supervenience sentence D which captures all the platonistic structure we're tempted to talk in terms of, we can write all our paraphrases using this), it promises to let us capture all desired inferential and explanatory relationships between, e.g., pure and applied mathematics the platonist can martial [19].

So the basic modal if-thenist paraphrase strategy above promises to let us take a satisfying modal-logicist perspective on many applied mathematical statements and explanatory hypotheses[20]. But how widely can the above paraphrase

[19] See 12.3.1 of [5]

[20] Some nominalists might worry about the above translations' use of mathematical vocabulary like 'set' and 'element' inside the $\Diamond$/$\Box$ of logical possibility/necessity. For, as stated, my paraphrases make claims about how it would be logically (not to say metaphysically!) possible for there to objects like sets with ur-elements. Nominalists who think 'set' is a meaningful predicate which just happens to have a necessarily empty extension, this is fine. However, nominalists who aren't fine with this should note that we can entirely banish terms like 'set'

strategy be applied? Can find a definable supervenience sentence D which would let us use it to nominalize all the mathematical explanations for scientific facts that have been used to make explanatory indispensability arguments?

4 Physical Magnitude Statements

A classic counting argument in Putnam's [23] raises a worry nominalistic paraphrase strategies cannot distinguish between (so as to imply different consequences regarding) infinitely many different possible values of physical magnitudes like mass, charge and length[21]

Field [10] (among others) this worry by noting that measurement theoretic uniqueness theorems suggests a solution to this problem — at least as regards the specific notion of length — if we accept substantivalism about space. Given some assumptions, which I'll call the claim that space is richly instantiated [22], we

and element from the above paraphrases, using any other first-order predicates and relations that don't occur in $\vec{N}$ instead. For example, we could uniformly replace 'set' and 'element' in the translation above with,'angel' and '...is transubstantiated into...' in our $T(\phi)$. This strategy is reminiscent of Putnam's strategy for stating potentialist set theory in [22].

[21] Putnam's counting argument in [23] notes that when formalizing a theory like Newton's law of gravity, the Platonist can appeal to notions like a mass relation, which relates physical objects to their mass in grams, or a mass ratio relation which relates pairs of objects to a number that's the ratio between their masses. Using these platonistic relations (relations to mathematical objects) they can distinguish –and write theories that imply different consequences given – infinitely many different possibilities (w.r.t. the length ratios), in a universe containing only two physical objects. In contrast, any nominalist paraphrase language (that only uses finitely many relations, which only relate physical objects), can only distinguish finitely many distinct possibilities for a world which contains only two physical objects. Accordingly, it seems that there couldn't possibly be any nominalistically acceptable theory which captures the full range of implications about objects standing in various different distance/length ratios which Platonist theories can distinguish.

In terms of the notions discussed above, this amounts to a suggestion that the above modal if thenist paraphrase strategy can't capture theories involving physical magnitude claims because no definable supervenience description D can pin down the expected behavior of Platonist physical magnitudes functions, like those attributing mass, charge and length in SI units (which can be used to draw the infinitely many distinctions mentioned above), given only the facts about how finitely many nominalistically acceptable relations apply.

[22] Specifically, we can prove the uniqueness claim above holds whenever the following three principles (which all happen to be stateable in the language of set theory with ur-elements) are satisfied. My presentation follows [24] in using the following principles.

Closure Under Multiples: Given a path x, there are paths y with lengths equal to any finite multiple of the length of x.

Archimedean Assumption: No path is infinite in length with respect to another, i.e., if $x \leq_L y$ then some finite can uniquely pick out the Platonist's intended length-in-meters function (from among all other functions from objects to real numbers) by saying it assigns length 1 to some canonical path and assigns lengths in a way that respects the following nominalistic relations:

- $\leq_L$ 'path p_1 is at least as long as path p_2'
- $\oplus_L$ 'the combined lengths of path p_1 and p_2 together are equal to the length of path p_3'. (I will say a function $l(x)$ respects $\leq_L, \oplus_L$ just if for all paths a, b and c $a \leq_L b \iff l(a) \leq l(b)$ and $\oplus_L(a, b, c) \iff l(a)+l(b) = l(c)$).

Thus, we have a formula ψ which picks out the Platonist's length-in-meters function at all possible words where length is richly instantiated[23]. So, at all such possible worlds, a scientific theory involving a length function $\phi(l)$ (in the language of set theory with ur-elements, with l being a name for this length function) will be true if and only iff the corresponding nominalist sentence $T(\phi)$ (below) is true.

> $T(\phi)$ $\Box_{\vec{N}}$ If there are objects satisfying our description of the hierarchy of sets with ur-elements $V_{\omega+\omega}$ then $(\exists f)(\psi(f) \wedge \phi[l/f])$'

Thus one might hope Platonist appeals to length relations can be harmlessly replaced by the strategy above. And maybe (as Field perhaps suggests in [10]) Platonist talk of mass, charge etc. functions could be handled similarly.

multiple of x is longer than y (i.e. there's a path shorter than y, which can be cut up into n segments each of which has the same length as x.

Relational Properties: The relations $\leq_L, \oplus_L$ have the basic properties you would expect from their role as length comparisons.

[23] At worlds where length isn't richly instantiated, the archimidean assumption needed for the measurement theoretic uniqueness theorem might not apply. So our description ψ might not pick out a unique correct length in meters function at such possible worlds.

4.1 Sparse Magnitudes Problem

However, a crucial difficulty, which I'll call the Sparse Magnitude problem threatens this proposal! For, although lengths are plausibly richly instantiated in our world, it's not clear that they're richly instantiated at all metaphysically possible worlds. And other physical magnitudes, like mass and charge, don't even seem to be richly instantiated in the actual world. Indeed, as Eddon puts it [8] (with slight adjustments to the choice of nominalistic primitives I've used above made in brackets):

> It seems possible for there to be a world, w_1, in which a and b are the only massive objects, and a is [three times] as massive as b. It also seems possible for there to be a world, w_2, in which a and b are the only massive objects, and a is [four] times as massive as b. Worlds w_1 and w_2 are exactly alike with respect to their patterns of [how the relations 'less than or equally massive' $o_1 \leq_M o_2$ and $\oplus_M(o_1, o_2, o_3)$ 'combined mass of a + mass of b = mass of c' apply]. And thus they are exactly alike with respect to the constraints these relations place on numerical assignments of mass. ... So it seems we cannot discriminate between the two possibilities we started out with.

These considerations threaten to block the above nominalist paraphrase strategy by showing that length is a special case. They suggest that other physical magnitudes (like mass) can't be pinned down in the same way that length can, and perhaps that the values of physical magnitudes doesn't supervene on facts about how *any* finite list of nominalistic relations apply[24]. Field notes and discusses a version of this problem in [11] the last chapter of[12].

[24]Thus a version of Putnam's famous counting argument in [23] threatens to re-arise, even for those nominalists like Field in [10] who avoid the specific concern about lengths he mentions by accepting the existence of spatial points or paths.

5 Cheap Tricks for Solving the Sparse Magnitudes Problem

Happily however, we can solve the sparse magnitudes problem by using two cheap tricks[5].

5.1 Four Place Relation

The first cheap trick is to note that if we (temporarily) assume that length is richly instantiated at all metaphysically possible worlds, we can solve the sparse magnitude problem by using length ratios to nominalistically pin down other physical magnitudes.

So, for example, we can uniquely pick out the Platonist's intended mass function (up to a choice of unit) by requiring that it assigns masses in a way that respects the following nominalistically acceptable four place relation between two objects (with mass) and two spatial paths:

- $M(p_1, p_2, o_1, o_2)$ which holds iff the ratio of the masses of o_1 to the mass of o_2 is $\leq$ the ratio of the length of path p_1 to the length of the path p_2.

That is, once we've pinned down an intended length function (up to multiplication by a constant), we can uniquely describe the intended mass in grams function m (in terms of its relationship to the length in meters function l) by requiring that it satisfies the following conditions [25].

- For all objects o_1 and o_2 and paths p_1 and p_2, $\frac{m(o_1)}{m(o_2)} \leq \frac{l(p_1)}{l(p_2)}$ iff $M(p_1, p_2, o_1, o_2)$

[25] For, note that any attempt to assign the wrong mass ratio r' to a pair of objects m_1, m_2 with mass ratio r can be ruled out by considering paths p_1, p_2 whose length ratio falls between that of r and r' and noting that $\mathscr{M}$ fails the above condition for a pair of paths such that $l(p_1)/l(p_2)$ falls between r and r'. And the existence of such a pair of paths is guaranteed by the assumption that length is richly instantiated (which, indeed, implies that length ratios are dense in $\mathbb{R}$).

- m assigns the value 1 to some canonical object we expect to have mass one gram.

In this way, we can write a definable supervenience sentence D which uniquely pins down intended behavior for length, mass and other such physical magnitude functions – at least at all possible worlds where length is richly instantiated.

5.2 Holism trick

But what about metaphysically possible worlds where length isn't richly instantiated? If there are such worlds then (for all I've said so far) our parapharse strategy might fail at them. But to produce a successful paraphrase (in the sense I've specified) we need a sentence $T(\phi)$ which the Platonist thinks has the same truth-value as ϕ at all metaphysically possible worlds[26].

One option would be to say space is metaphysically necessarily richly instantiated. If one accepts substantivalism about space, as I am doing for the purposes of this paper (much as Field does in [10]), there is some attraction to this assumption. However, certain trends in physics raise a worry about this. For, physicists do seem to consider hypotheses on which *space itself* is quantized, so that that length isn't richly instantiated (even from a substantivalist point of view, where spatial points and paths exist and hence can stand in length relations). And we might want to say this kind of epistemic legitimacy (quantized space not being ruled out a priori) suggests we should regard quantized space as a genuine metaphysical possibility. Thus even if we think that space is actually richly instantiated (as it seems to be), we might want to deny that length is metaphysically necessarily richly instantiated.

Happily, however, it turns out that we don't need to make any such controversial assumption. For the second cheap trick is to note the following things

[26] That is, the Platonist must acknowledge that for all metaphysically possible worlds w, ϕ is true at w iff $T(\phi)$ is true at w.

- Our best current physical theories imply that length is richly instantiated.
- the claim R that space is richly instantiated is easy to paraphrase using the basic modal-if thenist strategy above. For we can state it using only nominalistic notions $\leq_L, \oplus_L$ and platonistic notions (set theory with urelements) which we know how to write a definable supervenience sentence D to uniquely pin down.

Thus, we can create a nominalistic sentence which (the Platonist must think) has the same truth value as ϕ at *all* possible worlds. To quickly see why, consider the following modified paraphrase strategy.

$T^*(\phi)$: $T(\phi) \wedge T(R)$

At worlds where length is richly instantiated, the Platonist must allow that $T(\phi)$ has the same truth-value as ϕ by our argument, and T(R) is true (since it acceptably nominalizes the claim that length is richly instantiated), so the above conjunction will have the correct truth value. And at worlds where space isn't richly instantiated, the Platonist will say ϕ is false (since our best physical theories imply that space is richly instantiated). Our paraphrase $T^*(\phi)$ will also be false, since T(R) must be false because it adequately paraphrases the claim that space is richly instantiated. Thus, in both cases, our paraphrase has the intended truth value.

In fact it turns out that attractive and relatively uncontroversial modal reasoning (which I won't try to summarize here) [5] shows our basic paraphrase strategy $T(\phi)$ *already* secures intended truth-values at possible worlds where length isn't richly instantiated.

Thus it appears that by the above two cheap tricks, the modal if thenist can solve the sparse magnitude problems sufficiently well to answer the classic Quinean indispensability argument.

6 Baker's Rocketship

However one final challenge remains. (Although officially targeted against comparativism about physical magnitudes not nominalism about mathematical objects), Baker's rocketship argument in[2] highlights a further worry for the program above.

In this section, I'll briefly summarize Baker's core argument against comparativism in [2]. I will then note how Baker's considerations also challenge measurement theoretic uniqueness theroem based approaches to nominalizing physical magnitude statements – including the proposal above– by suggesting that scale matters. When logically regimenting physical theories like Newtonian mechanics — writing something which pins down intended values for all physical magnitudes up to multiplicaiton by a constant is not enough.

In my description of modal if thenism, I breezily talked about 'appealing to a canonical unit fixing objects' when fixing unique intended behavior of a mass in grams function –not just respect for ratio facts. If successful, this would allow our paraphrases to state exactly the kind of claims about specific values of physical magnitudes Baker's argument suggests we need to capture Newtonian Mechanics. But can this idea be realistically cashed out? Or can the modal nominalist pin down the intended scale of physical magnitude functions in some other way? At the end of the section, I will lay out a proposal to address such worries by making one small change to the paraphrase strategy proposed above.

6.1 Baker's Problem for Comparativists

Baker's arguments are explicitly intended to to target a view about the metaphysics of physical magnitudes called comparativism, which he evokes by quoting Dasgupta's [7] as follows

> [T]hings with mass stand in various determinate mass relationships

> with one another, such as x being more massive than y or x being twice as massive as y... [C]omparativism is the view that the fundamental facts about mass concern how material bodies are related in mass, and all other facts about mass hold in virtue of them. (Dasgupta, forthcoming, 1)[2]

> [T]he comparativist thinks that the fundamental, unexplained facts about mass are facts about the mass relationships between bodies, and all other facts about mass hold in virtue of those mass relationships. This leaves open what kinds of mass relations those fundamental facts concern: they might concern mass ratios such as an object being twice as massive as another, orderings such as an object being more massive than another, or even just linear structures such as an object lying between two others in mass. But this in-house dispute will not matter for our purposes.[7]

Exactly how to best formulate comparativism is somewhat controversial (e.g., Baker ultimately somewhat disagrees with Dasgupta's formulation)[27]. However I take the guiding intuition to be clear. And all that will matter for Baker's argument and our purposes in this paper (and something which I take all parties to agree on) is that comparativism implies the following claims

- Any two complete descriptions of a metaphysically possible world that differ only in that all the masses in one description are double (or any other positive multiple) those in the other actually refer to the same possible world.

[27] Baker proposes his own sharpened definition as follows.

> comparativism about some quantity – or family of quantities – is the view that the fundamental facts about those quantities are given by the scale-independent relations comparing different objects' values of the quantities. [2]

- Therefore, any physical theory which describes any metaphysically possible world must make the same prediction when all masses are multipled by the same positive constant.

Baker argues that comparativism is incompatible with (one powerful but intuitive version of) the claim that Newtonian Mechanics is deterministic. Specifically he appeals to the idea Newtonian Mechanics intuitively makes deterministic predictions about how *different physical magnitudes trade off against one another to determine physical outcomes* in various concrete cases[28]. For example it predicts

- The velocity required for a rocketship to escape a planet's gravitational pull given the planet's mass.
- The acceleration of a sliding hockey puck as a function of its mass and frictional forces.

And Baker points out that *which* outcomes Newtonian Mechanics predicts for such scenarios reflects the absolute values of certain physical magnitudes, not just intra-magnitude ratios. For example, there are certain pairs of mass m and velocities v such that, Newtonian mechanics predicts that[29]

- a rocket launched from the surface of a planet with mass m at velocity v in some general situation S1 will escape

[28]Technically there are reasons to think Newtonian Mechanics isn't exactly deterministic. But this doesn't matter to the argument. Although Baker and subsequent literature frames things in terms of capturing the intuitive determinism of Newtonian Mechanics, he could equally well made his point by talking about capturing the fact that intuitively Newtonian Mechanics requires a unique outcome for his imagined experiments with rocketships and sliding objects, and noting that comparativism has trouble doing this.

[29]Why? Newtonian mechanics says that the escape velocity for an object lanched from the surface of a planet with mass m and diameter d is
$v = \sqrt{\frac{2mG}{d}}$,
where G a constant (the gravity constant). Hence it predicts if the escape velocity for the earth is v, the escape velocity for launches from a planet with the same diameter and twice the mass is $\sqrt{2}v$.

- a rocket launched at velocity v from a planet with mass 2m in a situation S2, which is exactly analogous to S1 but with the masses of all objects (the rocket, the planet etc) doubled, would not escape

.

This poses a serious problem for the comparativist. For the comaparativist can't allow a difference between the two supposedly distinct world states mentioned above.

To make the intuitive problem here more explicit, we'd like to say that Newtonian mechanics is determinsitic in the following sense. There are facts (presumably physical magnitude facts) intrinsic to the state of the world at time t which combine with Newtonian Mechanics to determine a unique outcome for the rocket launch. So (Baker argues) we expect worlds where the principles of Newtonian Mechanics are physically necessary law to have the following Laplacian Determinism.

> **Laplacean Determinism**. A world w is deterministic iff, for any time t, there is only one physically possible world whose state at t is identical to w's[2].

However the comparativist cannot say this. For they only acknowledge physical magnitude ratio facts, which don't distinguish between S1 and S2. Hence they must allow that the single momentary world state at time t which is picked out by this description can evolve forward in two different ways (the rocket ship escaping or not escaping) that are equally compatible with Newtonian Mechanics.

6.2 Why My Nominalist Paraphrase Strategy Faces No Challenge From Baker's Examples

But what (if anything) does this mean for the project of nominalistically paraphrasing physical magnitude statements?

I think Baker's example raises a prima facie worry for all nominalistic paraphrase strategies make significant appeal to the measurement theoretic uniqueness theorems referenced above[30]. For no theory which only talks about intra-physical magnitude ratio relations (like the relations $\leq_L, \oplus_L$ mention above) will be able to distinguish between situations S1 and S2 above where we intuitively want to say Newtonian Mechanics requires different outcomes. So no such theory can logically regiment Newtonian Mechanics in a way that honors the Laplacian determinacy property above.

Now, theoretically, the cheap tricks proposal above tries to avoid this problem, by requiring that an intended mass in kg function do more than respect how the relations $(\leq_L, \oplus_L)$ needed for the measurement theoretic uniqueness theorem. I also required that such functions must assign value 1 to certain 'canonical unit-fixing objects'. Doing this lets us pin down unique intended behavior for a mass-in-grams and length-in-meters functions etc., and thus state claims which distinguish between S1 and S2. This lets us articulate a version of Newtonian Mechanics satisfying our intuitive Laplacian determinacy intuitions referenced above.

This provides one style of response to the above worry raised by Baker's rocketship example. However there are reasons to doubt that suitable unit fixing objects can always be found. So I will end this section by explaining this problem, and then proposing an alternate solution to the challenge of fixing precise intended scales for physical magnitude functions.

[30] c.f. [26]

6.3 More About How To Appeal To A Canonical Unit Fixing Object

For I think a problem arises when we start to think about what such unit fixing objects could be (e.g., a cube of water? the meter stick in Paris?). If we could be guaranteed that (as a matter of physical law) our chosen unit-fixing objects exists at all times, things would be fine. For my paraphrase strategy would indeed let us regiment Newtonian Mechanics in such a way that, for each time t, our nominalized theory combines with facts completely intrinsic to the state of the world at time t (rather than relations to some past, future or hypothetical unit fixing object), to require a unique outcome.

However, modern measurement systems do not generally seem to rely on such physically necessarily persisting unit fixing objects. Instead, SI units are defined in terms of fundamental physical laws and counterfactual experimental outcomes.

Moreover, such logical regimentations seem unhelpful for evaluating counterfactuals about scenarios in which there is no canonical meter stick, or the canonical meter stick has a different length.

Accordingly, I will now make a different proposal for how we can fix an intended scale for physical magnitude functions by appealing to finitely many nominalistically acceptable properties and relations:

- To pin down a unique intended length in meters function, introduce an atomic predicate 'is one meter long', whose meaning is given by connection to the results of counterfactual measurement procedures (if performed in the actual world) rather than reference to a fixed object like a meter stick.
  - Take this term (and the atomic two place relation term proposed below) to be rigid designators – just as an atomic predicate 'is one meter long' might be thought to rigidly designate a property which

can be referred to by appeal to the length of the canonical meter stick in the actual world.

- Have your supervenience description D fix units for length by requiring that an intended length in meters function must assign all spatial paths which have the 'one meter long' property the value 1.
- That is, pick out the intended length in meters function l, by requiring that for every spatial path p, $l(p) = 1$ iff p has this atomic property (i.e. p is 1 meter long), as well as that l respects the relations $\leq_L, \oplus_L$ (as discussed in the sections above).

- To pin down unique intended physical magnitude function with units (e.g., mass in kg), for each of the finitely many physical magnitudes that might occur in your theory, introduce a corresponding atomic three place relation like 'x is as massive in kg as the ratio between paths y and z', which serves as a bridge between different physical magnitudes.
  - The reference of these atomic relations can be communicated by appeal to more convoluted counterfactual measurement procedures, but again we should take them to rigidly designate.
  - Have your supervenience description D fix units by requiring that the intended mass in kg/brightness in lumens/etc. function assign values in a way that respects this ratio relation (when considered together with an intended length in meters function whose behavior we have uniquely pinned down as above).
  - That is, pick out the intended mass in kg function m by noting that it assigns masses in such a way that for all objects x and spatial paths y and z, $m(x) = l(y)/l(z)$ iff this atomic relation applies. $\forall x \forall y \forall z[m(x) = l(y)/l(z) \leftrightarrow$ 'the mass of x compares to 1 kg as

the length of path y to that of path z']

Fixing a unique intended extension for each physical magnitude function in this way lets us write a definable supervenience description D which indeed uniquely determines intended behavior for all physical magnitudes functions (at all possible worlds where length is richly instantiated and some physical path has length 1 meter[31]). Thus plugging this description D into the modal if-thenist paraphrase strategy above lets us write down a paraphrase of Newtonian mechanics which gets the possible worlds truth conditions right, from a Platonist point of view (as desired).

Indeed the above strategy for fixing scale lets us pin down all physical magnitudes at time t, using only nominalistic relations between objects (including spatial paths) existing at time t. Thus, it lets us nominalize Newtonian mechanics in a way that vividly satisfies the Laplacian determinacy intuition evoked by Baker above. For it lets us write down a version of Newtonian mechanics which combines with facts about properties and relations of objects at time t, to determine a unique outcome for experiments with rocketships and the like [32].

[31]Note that if these requirements are satisfied then for every value v some physical magnitude function with units could take on, there will be some pairs of spatial paths within the path of length one meter, whose ratio gets arbitrarily close to v.

[32]Technically, there is one further issues we need to capture, namely physical constants. The Platonist who accepts Newtonian Mechanics may say (if they regard physical constants as names that rigidly designate a certain real number which can only be learned by successive experiments on the assumption that Newtonian Mechanics qua schema is true) they have a finite statement of Newtonian Mechanics which, when combined with a complete description of the world at time t implies unique precise world state for every time after t. And they might challenge the nominalist to come up with a similar finite human-stateable version that combines with all the facts about a world state at some time t to metaphysically necessitate precise world states for all times after t. However, a nominalist who is willing to use inelegant atomic relations can rise to this challenge, for they can replace the Platonist's finitely many names which rigidly designate with finitely many atomic relations between spatial paths which rigidly designate, treating (in effect) 'path ... is [actual world gravity constant] times as long as path ...' as an atomic relation. Our description D of intended Platonist structure can then identify the gravity constant as, e.g., for any pair of paths which stand in this atomic relation a and b, the ratio between the number assigned to a and that assigned to b, by the intended length in meters function. Note that all worlds where length is richly instantiated, there will always be some pairs of spatial paths a and b which stand in this relation.

7 Conclusion

In this paper, I have argued that a mathematical nominalist (who accepts certain notions independently motivated by the literature on potentialist set theory) can plausibly answer both classic Quinean and explanatory indispensability worries raised by scientific use of physical magnitude statements by deploying certain cheap tricks[33].

While previous work established that nominalists could respond to indispensability arguments via cheap tricks, an important problem remained: how to fix a scale for physical magnitude functions in a way that preserves modal good behavior. This paper attempts to solve that problem, demonstrating that a nominalist-friendly approach can succeed without reliance on strong modal realism.

Thus I think one can plausibly use these cheap tricks to create a modal nominalist paraphrase which answer the core Quine-Putnam indispensability challenges concerning physical magnitudes [34] (for purposes like vindicating the possibility of taking a modal-logical perspective on all mathematics).

However, arguably, further important challenges remain. First, there's a reference worry. The cheap tricks proposed above can be used to paraphrase total physical theories (as needed to answer the Quinean challenge), but not more limited claims that, for example, do not include commitment to length being richly instantiated.

Second, there's a grounding and intrinsicality worry. The paraphrases I've

[33]However, I don't think this shows that all is plain sailing for the nominalist. For example, note that the four-place relations that I've invoked are not very metaphysically elegant, and hence are ill-suited to ground physical magnitude facts. Accordingly, something like a grounding indispensability worry may remain ('if there aren't numbers related to objects via a mass ratio-relation, what grounds mass facts?'), even if we can solve classic and explanatory indispensability arguments by logically regimenting our scientific theories involving physical magnitudes in the way I've suggested.

[34]I haven't said anything about the very important and realistically central case of probability statements, and how to paraphrase physical magnitude claims that apply to events.

mentioned above are crafted to get truth conditions right. But they do so by using conceptual machinery like the four place relation in §5, which is both inelegant (and therefore implausible as metaphysically fundamental ideology) and prone to introduce intuitively irrelevant extrinsic content into paraphrases. So even if my proposal succeeds in solving the Quinean and explanatory challenges at issue in this paper, arguably further work remains to be done regarding the following projects

- Field's ambition of stating physical laws *purely intrinsically*, rather than by talking about actual physical objects relationship to things like natural numbers[10].
- Sider's ambition of stating facts about fundamentalia (adequate to ground the truth of all other claims we believe) using *only maximally joint carving* vocabulary[25].

A Set Theoretic Mimicry

Although the conditional logical possibility operator is proposed as a conceptual and metaphysical primitive, we can use the familiar formal background of set theory to *mimic* intended truth conditions for statements in a language containing the logical possibility operator $\Diamond$ alongside usual first order logical vocabulary (where distinct relation symbols R_1 and R_2 always express distinct relations) as follows.

> A formula ψ is true relative to a model $\mathscr{M}$ ($\mathscr{M} \models \psi$) and an assignment ρ which takes the free variables in ψ to elements in the domain of $\mathscr{M}$[35] just if:

[35] Specifically: a partial function ρ from the collection of variables in the language of logical possibility to objects in $\mathscr{M}$, such that the domain of ρ is finite and includes (at least) all free variables in ψ

- $\psi = R^k_n(x_1 \ldots x_k)$ and $\mathscr{M} \models R^k_n(\rho(x_1), \ldots, \rho(x_k))$.
- $\psi = x = y$ and $\rho(x) = \rho(y)$.
- $\psi = \neg\phi$ and ϕ is not true relative to $\mathscr{M}, \rho$.
- $\psi = \phi \wedge \psi$ and both ϕ and ψ are true relative to $\mathscr{M}, \rho$.
- $\psi = \phi \vee \psi$ and either ϕ or ψ are true relative to $\mathscr{M}, \rho$.
- $\psi = \exists x \phi(x)$ and there is an assignment ρ' which extends ρ by assigning a value to an additional variable v not in ϕ and $\phi[x/v]$ is true relative to $\mathscr{M}, \rho'$[36].
- $\psi = \Diamond_{R_1 \ldots R_n} \phi$ and there is another model $\mathscr{M}'$ which assigns the same tuples to the extensions of $R_1 \ldots R_n$ as $\mathscr{M}$ and $\mathscr{M}' \models \phi$.[37]

Note that this means that $\perp$ is not true relative to any model $\mathscr{M}$ and assignment ρ.

If we ignore the possibility of sentences which demand something coherent but fail to have set models because their truth would require the existence of too many objects, we could then characterize logical possibility as follows:

> **Set Theoretic Approximation**: A sentence in the language of logical possibility is true (on some interpretation of the quantifier and atomic relation symbols of the language of logical possibility) iff it is true relative to a set theoretic model whose domain and extensions for atomic relations captures what objects there are and how these atomic relations actually apply (according to this interpretation) and the empty assignment function ρ.

[36]As usual (?) $\phi[x/v]$ substitutes v for x everywhere where x occurs free in ϕ

[37]As usual, I am taking $\Box$ to abbreviate $\neg\Diamond\neg$

References

[1] A. Baker. Are there Genuine Mathematical Explanations of Physical Phenomena? *Mind*, 2005.

[2] David John Baker. Some Consequences of Physics for the Comparative Metaphysics of Quantity. In Karen Bennett and Dean W. Zimmerman, editors, *Oxford Studies in Metaphysics Volume 12*, pages 75–112. Oxford University Press, 2020.

[3] Sharon Berry. Mathematical Access Worries and Accounting for Knowledge of Logical Coherence. *The Journal of Philosophy.* forthcoming.

[4] Sharon Berry. Modal Structuralism Simplified. *Canadian Journal of Philosophy*, 48(2):200–222, 2018.

[5] Sharon Berry. *A Logical Foundation for Potentialist Set Theory.* Cambridge University Press, Cambridge, 2022.

[6] George Boolos. Nominalist Platonism. *Philosophical Review*, 94(3):327–344, 1985.

[7] Shamik Dasgupta. Absolutism Vs Comparativism About Quantity. *Oxford Studies in Metaphysics*, 8:105–150, 2013.

[8] Maya Eddon. Quantitative Properties. *Philosophy Compass*, 8(7):633–645, 2013.

[9] John Etchemendy. *The Concept of Logical Consequence.* Harvard University Press, 1990.

[10] Hartry Field. *Science Without Numbers: A Defense of Nominalism.* Princeton University Press, 1980.

[11] Hartry Field. Is Mathematical Knowledge Just Logical Knowledge? *Philosophical Review*, 93(4):509–552, 1984. Publisher: Duke University Press.

[12] Hartry Field. *Realism, Mathematics & Modality*. Blackwell Oxford, 1989.

[13] Hartry H. Field. *Saving Truth from Paradox*. Oxford University Press, March 2008. Published: Paperback.

[14] Mario Gómez-Torrente. A Note on Formality and Logical Consequence. *Journal of Philosophical Logic*, 29(5):529–539, October 2000.

[15] William H. Hanson. Actuality, Necessity, and Logical Truth. *Philosophical Studies*, 130(3):437–459, 2006.

[16] Geoffrey Hellman. *Mathematics Without Numbers*. Oxford University Press, USA, 1994.

[17] Geoffrey Hellman. Structuralism Without Structures. *Philosophia Mathematica*, 4(2):100–123, 1996.

[18] Marc Lange. What Makes a Scientific Explanation Distinctively Mathematical? *British Journal for the Philosophy of Science*, 64(3):485–511, 2013.

[19] Øystein Linnebo. The Potential Hierarchy of Sets. *Review of Symbolic Logic*, 6(2):205–228, 2013.

[20] Øystein Linnebo. Putnam on mathematics as modal logic. In Geoffrey Hellman and Roy T. Cook, editors, *Putnam on Mathematics and Logic*. Springer Verlag, Berlin, 2018.

[21] Charles Parsons. *Mathematical Thought and Its Objects*. Cambridge University Press, December 2007. Published: Hardcover.

[22] Hilary Putnam. Mathematics Without Foundations. *Journal of Philosophy*, 64(1):5–22, 1967.

[23] Hilary Putnam. *Philosophy of Logic*. London: Allen & Unwin, 1971.

[24] Theodore Sider. A Crash Course on Measurement Theory.

[25] Theodore Sider. *Writing the Book of the World*. Oxford University Press, 2011.

[26] Theodore Sider. *The Tools of Metaphysics and the Metaphysics of Science*. Oxford University Press, 2020. Google-Books-ID: Gd7QDwAAQBAJ.