Which Sigma Algebra? A Challenge for Simple Realists about Objective Probability

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[Work in Progress: Please drop me a line if you're interested in discussing this or seeing the latest version]

Abstract

There seem to be facts about objective physical possibility. What I will call **simple anti-Humean realism** about objective probabilities (SAR) takes these facts to be relatively primitive and fundamental. Because simple anti-Humean realism takes probability talk at face value, it can seem like the natural position to take absent some conflicting larger philosophical project (e.g., empiricism, materialism, ambitions for extreme conceptual parsimony). But, I will argue that it faces an interesting internal challenge about how to explain why certain propositions lack objective probabilities.

1 Introduction

There appear to be facts about objective physical possibility. For example modern physics famously predicts that particles, when prepared in specific ways, have objective probabilities of producing certain measurement outcomes. Some philosophers attempt to reductively analyze claims about objective probability, grounding them in other concepts, such as patterns within a Humean mosaic of actual events [7].

In contrast, the default naive realist perspective which I will call **Simple Anti-Humean Realism** (SAR) rejects such reductive analyses. SAR posits that objective probability facts are not derived from, or reducible to, other kinds of facts. Instead, it asserts that fundamental reality includes primitive and irreducible modal facts about objective physical probabilities, akin to the way some theories postulate fundamental modal facts about logical, physical, or metaphysical possibility.

Because simple anti-Humean realism takes probability talk at face value, it can seem like the natural position to take absent some conflicting larger philosophical project (e.g., empiricism, materialism, ambitions for extreme conceptual parsimony). It may gain support from the large literature on apparent problems for more ambitiously reductive approaches to probability (like David Lewis' Humean analysis [7]). However, I will argue that simple anti-Humean realism about objective probabilities faces an interesting internal challenge – about how to explain why certain propositions lack probabilities.

In §2 I'll review how a certain kind of well known thought experiment involving physical hypotheses associated with non-measurable sets, suggests some propositions can lack objective probabilities.

In §3 I'll argue that allowing some propositions to lack objective probabilities raises a prima facie problem/explanatory challenge for the simple anti-Humean realist. For how could reality play favorites and treat some propositions as special in this way (i.e., in having objective probabilities while others lack them)? In §4, I'll develop and assess what I take to be the most obvious and best hope for answering this challenge, noting choice points and places where significant further philosophical work seems needed. Finally in §5, I'll consider whether simple antihumean realists can sidestep this explanatory challenge by rejecting some step in the initial argument that some propositions can lack objective probabilities.

2 Probability Gaps

As noted above, there seem to be facts about objective physical probability. But which things have objective probability?

We seem to assign probabilities to something like coarsely individuated propositions (e.g., sets of possible worlds, or events qua sets of possible outcomes for an experiment) perhaps considered relative to a point in time[1]. For whenever two claims have the same possible worlds truth conditions – i.e., are true in the same set of metaphysically possible worlds – we expect them to have the same objective probability[2].

So, at first glance one might expect *all* sets of possible worlds to have objective probabilities (with propositions incompatible with the history of the world up to some time t having objective probability zero relative to that time) and accept the following principles.

Objective Probabilities for All Sets of Possibilities:

- For every proposition (qua set of possible worlds) P and time t, there's an objective probability "at" time t, that P will wind up being true.
- For every set S of possible outcomes to an experiment, there's an objective probability (at the time of the commencement of

[1]To motivate relativization to (something like) a time, imagine a case where three coins are flipped in sequence and all come up heads. We might say the following. At the time of the first coin being flipped, the objective probability of all flipped coins coming up heads is 1/8th. However when the second coin is flipped (since the first coin has already come up heads), the probability that all coins will come up heads is 1/4.

In what follows I will sometimes talk about the probability of an event relative at a possible world and time. But I don't mean to a position on exactly *what* objective physical probabilities should be regarded as relative to, as this question seems unrelated to the puzzle proposed in this paper.

[2]For example, we think that the objective probability (at launch) that a rocket will reach Hesperus must be the same as the objective physical probability that this rocket will reach Phosphorous. And we take the current objective probability that global temperatures will rise more than 3 degrees to be the same as the current objective physical probability that global temperatures will rise more than 3 degrees and all triangles are trilaterals.

the experiment) that some outcome in set S occurs.

However there are actually powerful reasons for denying the above principles, which we can see from reviewing the following classic thought experiment.

2.1 Motivating Example

Imagine a dart that is intuitively ‘equally likely to land anywhere’ on a rectangular dartboard, which spans the interval [-1,2] (with distances measured in meters from an arbitrary zero point). For instance, the dart is equally likely to land in any two regions of equal area. Consequently, it has a probability of 1/3rd of landing in the middle third of the dartboard, corresponding to the interval [0,1].

Vitali proved the following claim (from the standard ZFC axioms of set theory) [3]. There is a countably infinite list of sets of real numbers $v_1, v_2, v_3, \ldots$ (call these the Vitali sets), which are disjoint, contained in the region [-1,2], jointly include all points in [0,1] and have the following property.

- Each set of numbers v_i can be produced by shifting the first set of numbers v_1 over by some amount. That is, for each region v_i, there's a real number q_i such that $v_i = \{y|y = x + q_i$ for some x in $v_1\}$.

So, intuitively (by appeal to a kind of translation invariance intuition that shifting a region over on the dartboard should not make a difference to the probability that a dart with the properties described above lands in that region[3]), we would expect the dart to have equal probability of landing in the region of

[3]One might try to express this intuition by saying that in the intuitively metaphysically possible informally described situation above, the probability of a dart landing in any region must be proportional to the area of that region, and areas are preserved by shifting all points in a region by a constant distance. But as we will discuss below, Vitali sets are have no Lebesgue measure and arguably physical regions corresponding to them should not be thought of as having areas.

physical space corresponding to any one of the Vitali sets v_i. Now consider the following proposition.

> U: The dart will land somewhere in the union of the spatial regions corresponding to $v_1, v_2, ...$

What probability can U have? By countable additivity (one of the standard axioms of probability), the probability of the dart landing somewhere in the union of the disjoint regions corresponding to the countably infinitely many disjoint Vitali sets should be the sum of the probabilities that it lands in each specific region corresponding to some v_i. But we expect the dart to have equal probability of landing in any such region. So the dart's probability of landing in the union of all these disjoint regions should be either 0 (if the probability of it landing in v_1 is 0) or infinite (if the probability of landing on v_1 is ≥ 0). The latter scenario is clearly impossible. So the probability of the dart landing in the union of the Vitali sets must be 0.

However, above we noted that the union of the physical regions corresponding to Vitali sets includes all points in the interval [0,1]. And we said the dart has probability $\frac{1}{3}$ of landing in this interval (since it makes up a third of the total region corresponding to [-1,2], which is occupied by the dart board). So, by monotonicity (a consequence of the standard probability axioms), the probability of U (i.e., the dart landing somewhere in the union of the revions given by the the Vitali sets), must be $\geq \frac{1}{3}$. This contradicts the previous argument that U has probability 0.

So there seem to be cogent, and indeed metaphysically possible, scenarios where a proposition qua sets of possible worlds (and some sets of possible outcomes) can't be assigned objective probability in any reasonable way[4]. Thought experiments involving (physical possibilities associated with) non-measurable

[4] I will survey some options for resisting this conclusion in §5

sets, like our story about the darts[5], suggest the following moral. We shouldn't assume that every function correctly assigning objective probabilities to *some* sets of physically possible outcomes (e.g., those where the dart lands in some physical region we can pick out in a more normal way) can be extended to one that assigns correct objective probabilities to *all* sets of physically possible outcomes.

Accordingly, standard (Kolmogorov) approaches to probability consider a probability triple which consists of a sample space, Ω, "which is the set of all possible outcomes for some random process or 'experiment' being modeled" together with an event space, which is "a set of events, F , an event being a set of outcomes in the sample space." and a probability function P which is only requried to assign probabilities to all the events *in this event space* rather than something like *all sets of possible outcomes* (i.e., all subsets of the sample space Ω). And a probability function can satisfy all the standard (Kolmogorov) probability axioms without assigning probabilities to all subsets of its sample space Ω. The event space (i.e., the collection of sets of possible outcomes assigned which *do* get assigned probabilities) doesn't need to be the powerset of the outcome space. It just needs to form a σ algebra for the set of possible outcomes[6].

A representative textbook presentation puts connection between thought experiments involving non-measurable sets and the fact that modern foundations of probability allow such limited event spaces in the following suggestive but breezy way.

> "Not every subset of the sample space Ω must necessarily be considered an event: some of the subsets are simply not of interest, others

[5]The Vitali sets are influential examples of non-measurable sets. The Lebesgue measure is the unique translation invariant measure on the reals assigning the unit interval measure 1.[1]. An exact parallel to the reasoning above (but applied directly to sets of numbers not sets of points) shows the Vitali sets cannot have a Lebesgue measure.

[6]That is, F just needs to be a subset of the $P(\Omega)$ which contains Ω and is closed under complement and countable union.

> cannot be 'measured'. ...[C]onsider javelin throw lengths, where the events typically are intervals like 'between 60 and 65 meters' and unions of such intervals, but not sets like the 'irrational numbers between 60 and 65 meters'.[**?**]"

(As we will discuss below) one might like a bit more explanation about what it means to say that a *sets of possible outcomes* (as opposed to, e.g., a set of numbers) 'can't be measured' and therefore cannot be assigned a probability. However, I take the basic point to be clear. There are powerful reasons for thinking perfectly good propositions (qua sets of possible worlds/outcomes) can lack objective (and, for that matter, actual and ideal subjective) probabilities[7].

So now we can return to the philosophy of probability and ask: can a simple anti-humean realist attractively accommodate this conclusion about objective probability (or somehow resist it)?

3 A Problem about Philosophical Sense of Probability Gaps for the SAR?

In this section I will explain why I think that (although an analogous move involving subjective probabilities might be unproblematic) recognizing that some some propositions lack *objective* probabilities creates a prima facie philosophical problem for the simple anti-humean realist. This problem concerns how to explain (or allow room for a possible explanation of) why some propositions have meaningful probabilities while others lack them.

In the case of subjective probabilities, it might be reasonably easy to give a satisfying answer to the above question. For one could say that we don't (and

[7] I take the above thought experiment to suggest that that some propositions can lack ideal subjective probability (as there is plausibly no probability which an ideal agent knowing only the facts about the dart stipulated above should assign to that dart landing in the region corresponding to Vitali set v1.

shouldn't) assign probabilities to sets of possible outcomes whose strange and gerrymandered nature prevents us from thinking or speaking of them. Vitali uses (and needs[8]) the axiom of choice to argue for the existence of Vitali sets $v_1, \dots v_n$ with the properties above, rather than concretely defining the Vitali sets by specifying membership conditions for them. So plausibly the kinds of weird propositions (described by appeal to Vitali sets) our thought experiment suggests lack probabilities are ones we have independent reason for supposing cannot be entertained and therefore cannot (and ought not) be assigned subjective probabilities.

Relatedly, there's nothing immediately puzzling about accepting that some *good human hypotheses about physics* aren't fully opinionated and therefore fail to assign a probability to certain sets of physically possible outcomes like the Vitali sets[9].

However when we turn to the question of why certian propositions lack *objective probabilities*, we seem to face a more daunting explanatory challenge.

For it can seem odd to suppose that reality itself just brutely favors certain sets of possible worlds/outcomes over others (by allowing objective probabilities to attach to them). And (prima facie, especially if we take talk of objective probability at face value, as the simple anti-humean realist does) the fact that *it would be difficult for humans to pick out or think about* certain propositions has no relevance to, and hence no ability to explain, why those propositions lack objective probabilities.

Now perhaps some alternatives to simple anti-humean realism about objective probabilities have an easy answer to the above question (why do some propositions lack objective probabilities?). For example, perhaps 'best system'

[8]See the discussion of [2] in §5 below.

[9]Intuitively a good, true, illuminating etc. physical theory could fail to imply a specific objective probability for some propositions in some scenario it classes as physically possible (even when combined with a precise and complete description of non-probability facts about that scenario).

Humean approaches can exploit the special conneciton they see between objective physical probabilities and things like good theories and subjective probabilities to answer our challenge as follows.

In general, Humean approaches to probability maintain that all fundamental facts about a possible world concern what actually happens, and take things modal facts about things like probabilities and physical law to be real but non-fundamental, grounded in (and explained by appeal to) facts about the actual history of the world[10]. And best systems approaches specifically attempt to ground/explain the truth of such modal claims by appeal to their being consequences of whatever scientific theory best systematize the actual history of the world. As Lewis puts it in [7]

> The virtues of simplicity, strength and fit trade off. The best system is the system that gets the best balance of all three. ...[T]he laws are those regularities that are theorems of the best system. But now some of the laws are probabilistic. So now we can analyze chance: the chances are what the probabilistic laws of the best system say they are.[11] [12]

So a best systems theorist might explain why some propositions lack objective probabilities by saying the following. All there is to being an objective probability is being implied by the best theory of the pattern of actual events. And there is no obvious reason why we should expect the *best* theory (in the sense of best combining simplicity, strength and fit referenced above) to be fully

[10]So (they maintain) there can't be two distinct possible worlds w and w', which agree in their total histories but differ regarding things like: their physical laws, counterfactuals or the objective probability of certain events occurring.

[11]Thus the objective physical probability at time t of some event occurring, is (something like) the probability assigned to this event by combining facts about the actual state of the world up to t with the statement of probabilistic laws which best systematizes the total actual history of the world (in the sense above).

[12]Such a view might complement proposals in the literature on Borel's paradox, that events only have conditional probability relative to a Fregean mode of presentation[10]/ a choice of a σ algebra partitioning up all the possible outcomes into distinct events[4].

opinionated. So we shouldn't expect all propositions to have probabilities.

In contrast, the simple anti-Humean realist sees objective probabilities as fundamental aspects of physical reality in a way that blocks such solutions — and so and leaves us with a serious prima facie challenge about explaining why certain meaningful propositions lack objective probabilities. If we are simple anti-Humean realists, saying that only some propositions have objective probabilities seems to require that the natural world/metaphysics itself (rather than human reasoning and observational faculties, or some theory chosen to best balance informativeness with concision) favors certain propositions (e.g., 'the dart will land in the region [0,1]') over others (e.g., 'the dart will land in region corresponding to the Vitali set v_3')[13]. But what could explain the fact that some propositions are so favored?

Perhaps the best systems theorist has a reasonable hope of answering the above question, but it's not clear that the simple anti-Humean realist does.

4 The obvious explanation?

In this section I will develop and note some challenges for what I take to be the most obvious and appealing way for a simple anti-humean realist (who accepts the above argument that not all propositions have probabilities) to respond to the resulting challenge/explanatory demand.

The strategy I have in mind develops the breezy suggestion quoted above, above that certain sets of possible outcomes lack objective probabilities because they 'cannot be measured'. It's not immediately obvious what it means to say that a set of possible worlds and outcomes (as opposed to e.g., a set of numbers) is non-measurable. But I take the intended proposal to be as follows.

[13]One might compare this intuition that facts about human graspability/possible reference are the wrong kind of things for fundamental laws of physics or metaphysics assigning probabilities to care about to common resistance to physical theories on which 'observation' collapses the wave function.

1. Certain *sets of numbers* (e.g. Vitali sets) are not measurable in the familiar sense of having no Lebesgue measure[14].

2. The fact that these sets of numbers are non-measurable explains why 'corresponding' *regions of physical space* (e.g. those which could be most conveniently defined by reference to these sets using commonplace vocabulary for length and other physical magnitudes) lack a well defined length/area/volume.

3. The fact that these regions of physical space lack a well defined length/area/volume somehow prevents 'corresponding' propositions from having well defined probabilities. For example, the fact that some region R lacks a well defined area might somehow ensure the proposition 'the next dart thrown will land in region R' either [15]:

   - lacks an objective probability at all possible worlds

   - lacks an objective probability at all possible worlds where the dart in question is equally likely to land in any two portions of a given area wherevere these portions have well defined and equal area.

4. All propositions (qua sets of possible worlds) have objective probabilities, except for those which 'can't' have objective probabilities due to something like the nonmeasurability considerations just referenced.

I think this strategy is reasonably attractive, but I want to note some places where philosophical work needs to be done to develop it.

First, we'd need to spell out the idea that sets of points in physical space 'correspond' to non Lebesgue-measurable sets of (n-tupples of) numbers, refer-

[14]The Lebesgue measure is the unique translation invariant measure on the reals assigning the unit interval measure 1.[1]

[15]Perhaps we might imagine these regions characterized in terms of Cartesian coordinates from the speaker.

enced in step one[16].

Second, we'd need to decide between the two options referenced in step three. It would be appealingly simple to say (as per the first bullet point) that there's something intrinsic to the proposition 'The dart will land in the region corresponding to the Vitali set v_i' (likely involving our need to reference non-measurable sets and regions of physical space when describing it) which makes the set of possible worlds where this proposition is true intrinsically 'unmeasurable' and thus unsuited to having an objective probability relative to any point in the history of any metaphysically possible world.

However this approach has counterintuitive consequences. For it requires us to say that the above propositions lack objective probability in all possible worlds and times. And this conflicts with the common intuition that it's fully conceivable and metaphysically possible for there to be a dart which *does* have a definite objective probability (say 1/2) of landing in the spatial region corresponding the to the Vitali set v_1.

I take the metaphysical possibility of such a dart to be further supported by common ideas relating metaphysical possibility to re-combination (expressed in Hume's treatise[6] and Lewis[8]) and the fact that one can easily tell a mathematically cogent story – obeying all the probability axioms – about what the objective probabilities of this dart landing in many other regions might be[17].

Accordingly one might prefer to say (as per the second bullet point for step three) that propositions/sets of possible outcomes corresponding to non-

[16]This should not be hard to do if we can help ourselves to a notion of an appropriate coordinate system for space.

[17]Basic Humean recombination intuitions about metaphysical possibility suggest the metaphysical possibility of a world where a dart (or at least a teleporting point particle) has the following different propensities: probability 1/2 of landing in the region corresponding to v_1, probability 1/4 of landing in the region corresponding to v_2, probability 1/8th of landing in the region corresponding to v_3 and generally probability $\frac{1}{2^n}$) to the region corresponding to v_n. And we can further assign probabilities to a σ algebra of events which includes (all and only) the empty set of possible worlds and the sets of possible worlds corresponding to arbitrary unions of these countably many basic/atomic events in the obvious way, while satisfying the standard probability axioms.

measurable sets have objective probabilities of being realized relative to some possible worlds, while lacking them relative to others[18]. In this case, it can be a metaphysically contingent (and presumably empirical) question whether the claim that some dart will land in a certain spatial region has an objective probability. I don't think the resulting view is completely unpromising. However, I do want to highlight how it leaves us with more philosophical work to do in saying *what features of a possible world* allow/prevent propositions corresponding to non-measurable sets in the sense above from having objective probabilities and why. Also this approach requires (somewhat) sacrificing the the idea that non-measurability alone explains why certain propositions lack probabilities.

A third and final challenge for developing the story above concerns fleshing out step four – i.e., the maximalist idea that all propositions have objective probabilities except those which somehow 'can't' on pain of paradox/violating some core principles of set theory or probability. We've seen one kind of conflict which could prevent one from assigning a proposition a definite objective probability above. But could there be others? Which axioms count as inviolable core principles for these purposes?

Overall, I claim simple anti-Humean realists about objective physical probabilities have (at least) further philosophical work to do, if they want to explain which propositions lack objective probabilities by implementing the strategy sketched above.

[18]For the reasons described above, we might also relativize objective probability attributions to something like times in the history of the world or points along some foliation.

5 Resisting the Argument that Some Propositions Lack Probabilities

Finally, an alternative strategy for the simple anti-Humean realist would be to avoid my challenge by rejecting the argument in §2.1 that some propositions (could) lack objective probabilities. I will end this paper by surveying what I take to be the best options for resisting this argument, and explaining why I don't think they are very promising.

First, you could deny that my description of a scenario where a dart is 'equally likely to land anywhere' (and hence, e.g., equally likely to land in any two regions of equal and well defined area) on a rectangular dartboard is coherent or expresses something genuinely metaphysically possible[19]. But this seems clearly unintuitive.

Second, you could reject the axiom of choice or some other premises used in Vitali's pure mathematical argument for the existence of sets of numbers with the properties claimed in §2.1. As noted above, the Vitali sets v_i are not Lebesgue measurable. And Solovay showed we need the axiom of choice to establish the existence of any (Lebesgue) non-measurable set (i.e., that there are structures which satisfy all the other ZF axioms of traditional set theory and satisfy the claim that every set has a Lesbegue measure)[2]. However, rejecting one of the standard ZFC axioms of set theory is clearly a boldly revisionary move.

Third – and perhaps more initially attractively– you could say the above description of the dart board is cogent (and corresponds to something genuinely metaphysically possible) but reject the idea that, since each Vitali set is got by uniformly shifting v_1 over by some amount, the dart must have equal proba-

[19] Perhaps one could motivate this view by referencing the Borel paradox, about how to assign *conditional* probabilities to claims about a dart that's stipulated to be equally likely to anywhere on a sphere (in the same intuitive sense), as discussed in Esawaran's [4].

bility of landing in the physical region corresponding to each Vitali set v_i. As noted above, the Vitali sets are not measurable and you might think the above intuitions about uniform shifting over implicitly invokes a concept of "area" that only applies to measurable regions. So you could, in principle, reconcile the metaphysical possibility of our dart scenario with the expectation that all sets of possible outcomes/propositions have objective probabilities by saying the dart has

- equal probability of landing in regions of the dartboard with well defined and identical areas
- different probability of landing in the spatial regions corresponding to different Vitali sets v_i (which lack well defined areas) – with the sum of these countably infinitely many probabilities being somewhere between $\frac{1}{3}$ and 1, as desired.

However, it is difficult to imagine fleshing out this approach with any plausible story about which regions the dart is more likely to land in and why. For example, any (putatively) necessary and a priori view about of the regions corresponding to different Vitali sets a dart satisfying our initial description of a scenario above would be more likely to land in seems arbitrary and hard to motivate[20]. Thus this move threatens to replace one challenge (why do some propositions have probabilities while others don't?) with another (why is the

[20]It might be better to say the above description of the dart's behavior is cogent but incomplete: realizable in different ways coresponding to different choices for how to assign different objective probability to the the dart landing in regions of space corresponding to the different Vitali sets. Perhaps one can motivate this view by comparing our dart scenario to Thomson's lamp[9]. One might say the question of what probability a dart with the properties described in §2.1 above has of landing in each of the regions v_i is analogous to the question of what position a light switch will end in if it starts in the "on" position and gets flicked on and off infinitely many changes within a finite interval. In both cases a description seems like it must settle the answer to a follow up question -but this description turns out to be satisfiable in different ways that yield different answers to the follow up question. However, accepting that darts which are both 'equally likely to land anywhere in the rectangle' in the intuitive sense initially evoked can differ in their probability that the dart lands in a certain region v_i still feels a bit counterintuitive.

dart more likely to land in some regions than others?).

Fourth, you could reject countable additivity – or some other of the standard axioms of probability we used to derive paradox from the assumption that the dart has a well defined and equal probability of landing in a physical region corresponding to each v_i. Some independent reasons for concern about countable additivity have been proposed[21], but I take the motivations for sticking with such an intuitive and well entrenched principle to be clear.

So, overall, I don't see any easy way for a simple realist about objective probabilities to resist the conclusion that some propositions (qua sets of possible worlds) lack objective probabilities. But, as we saw, above accepting this claim raises some immediate questions and challenges for the simple realist.

6 Conclusion

In this short paper I have highlighted a tension between simple anti-Humean realism about objective probability and a familiar argument that certain propositions (qua sets of possible worlds) must lack definite probabilities. I've suggested that this tension provides an internal challenge for simple realists about objective probabilities. I think considering this issue may have philosophical consequences in two ways.

First, defenders of simple realism about objective probabilities could respond to this challenge by providing a principled account of which propositions lack objective probabilities and why. I have reviewed what I take to be the most obvious and natural strategy for doing this and highlighted some initial (not to say unsolvable) problems for that approach.

Second, opponents of simple ant-Humean realism about objective probability might try use the challenge above to further motivate a non-face value reading of

[21] See the summary of such concerns at the beginning of [5].

the logical structure or grounding facts relevant to objective probability. They might argue that that such approaches (e.g., best systems theories) can better explain why some propositions lack probabilities.

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