QUANTIFIER VARIANCE AND EKLUND'S CRITICISMS

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[Note: this is very much a work in progress!]

1. Introduction

Philosophers of mathematics have been much struck by mathematicians' apparent freedom to introduce new kinds of mathematical objects, such as complex numbers, sets and the objects and arrows of category theory. Mathematicians seem to willingly accept existence claims about new kinds of mathematical objects on the basis of considerations that look much more like arguments for the internal consistency or coherence of mathematical objects than the arguments which are required to establish the existence of new fundamental physical particles or species of animal.

Quantifier variance provides one popular way to account for this ontological profligacy while maintaining the intuitive realist idea that numbers and sets literally exist in the same sense as any other kind of object. Quantifier variance is a view in metaontology which holds that there are a range of similar but distinct senses available for the existential quantifier to take on in different languages. If one accepts quantifier variance then one can think about mathematicians' knowledge of basic foundational claims characterizing new kinds of mathematical objects as follows. When mathematicians introduce a new kind of mathematical object they accept a collection of existence statements S characterizing the intended structure of some new kind of mathematical objects (sets, irrational numbers, categories). In doing this they are, in effect, making a kind of implicit stipulative definition. This stipulative definition systematically shifts the meaning of the quantifier in their idiolect in such a way as to ensure that all the sentences in S express a truth, without altering the truth value of normal statements in the language.

Unfortunately however, quantifier variance faces a number of serious objections. First, there is a variant of the bad company argument. The quantifier variantist[1] says that mathematicians can reliably form true beliefs by making certain kinds of stipulations characterizing the intended structure of the numbers and the sets. However we must also admit that making other kinds of stipulations, like the logically inconsistent stipulations characterizing Frege's extensions or naive set theory, is *not* a reliable way of forming true beliefs. Thus, the proponent of ontologically inflationary stipulations is pushed to explain why some stipulations are acceptable and others are not.

Second, there is an objection raised by Matti Eklund[2] about how to make sense of the apparent logical and compositional structure of languages more ontologically profligate than one's own. Plausibly all the reasons we currently have for thinking our own language must have logical and compositional structure also apply to alternative idiolects which we could adopt via making stipulations which introduce new kinds of mathematical objects. Thus it is attractive to think that truths in this language can have the same kind of logical structure as truths in our own language and contain true statements of the form $F(c)$ even when the current language doesn't recognize an object for c to refer to. But how can we express or understand the truth of such logically structured claims without incurring commitment to the existence of objects to be referred to by expressions in this more profligate language?

[1]I mean here the quantifier variantist who wants to use quantifier variance to account for mathematicians apparently easy introduction of new kinds of mathematical objects mentioned above. Philosophers like Hirsch [CITE] who advocate quantifier variance for other reasons need not make this claim.

[2]CITE

In this paper I will propose a general picture of the nature and range of variant senses available for the quantifier to take on in languages similar to ours, and then show how this picture allows us to give a principled response to both of the challenges above.

2. Motivating Quantifier Variance

Let me begin by doing a little to motivate the quantifier variance approach to understanding mathematicians' freedom to introduce new kinds of mathematical objects.

Consider our (apparent) knowledge of the following three claims: there is a first natural number, every pair of reals a and b gives rise to a unique complex number of the form $a + bi$, any sofa which is perforated such-and-such has two holes in it. Each claim in the list above is ontologically inflationary in the sense that they license us to go from knowledge of objects of some kinds (the reals π and 3, a sofa which is perforated in certain ways) to the assertion that another object of a new kind (natural number, complex number, hole) exists (the complex number $\pi+3i$, a hole in the sofa). The first claim directly commits us to the existence of an object, the first natural number, however this can be understood as the limiting case of an ontologically inflationary stipulation.

In view of the ontologically inflationary character of the three statements above, one might think a fairly substantive process of investigation would be required to establish their truth. However, if you actually ask yourself what justifies your belief that there is a unique complex number of the form $a + bi$ for each pair (a, b) of reals, or that any possible world in which a sofa is perforated in a certain way will also count as containing a number of holes it is tempting to say something quite different, like that this is "just part of the definition of complex number" or "just part of what you mean by hole."

This kind of intuitive triviality and appeal to language suggests the following daring proposal with regard to the epistemology of ontologically inflationary claims like the ones listed above. Perhaps claims like those above can be known via a reliable process of stipulative definition. Think of making (acceptable) stipulations as a process on par with observation and deduction which reliably produces true beliefs, and if all goes well, knowledge. Observation reliably produces true beliefs by causally connecting our acceptance or rejection of a proposition to facts about the world which this proposition makes claims about. In contrast, acceptable stipulations reliably produce true beliefs by getting us to accept certain sentences and then shifting our language in such a way as to ensure that these sentences will express a true proposition. Presumably the terms hole and number were not actually introduced by a ritual utterance of "I stipulate that..." but perhaps our frequent use of (and acceptance without reference to other kinds of justification) claims like those mentioned at the beginning of this section functions as a stipulative definition.

If there are a range of similar but slightly different senses for the quantifiers to take on, then perhaps ontologically committal stipulations have the power to be reliable sources of true belief by forcing the quantifiers and the relevant terms (hole, number etc.) to take on new meanings in a way that leaves the truth conditions of commonly used sentences in the initial language unaltered but causes the stipulated principles to come out true. If this is right, then our knowledge of certain basic ontologically inflationary claims like the principles above may be best understood as the fruit of a process of stipulation, which left us both speaking our current language and truly believing whatever propositions are expressed by certain (stipulated) claims.

3. The Two Stage Approach to Existence

Now that we have said something about the appeal of the quantifier variance proposal, let us turn to the details of how I propose to spell it out. My key idea is to think of existence statements in our language as being made true by the world in a less direct fashion than one might otherwise have supposed. At least two broad pictures of how existential sentences are made true by the world are possible.

On what I take to be the standard picture, existential statements are made true by latching on to parts of the world fairly directly. The world already comes divided up into different objects, and the only thing that can vary between different languages (or idiolects or contexts) is what quantifier restrictions are in play[3]. If we think of the truth or falsity of whole sentences being determined compositionally via Tarskian[4] rules then we should think of the domain of objects which the Tarskian rules apply to as being supplied directly by the world and then restricted in a context sensitive fashion.

In contrast, I would like to advocate an alternative approach takes existence facts to be determined more indirectly. Each language is associated with a collection of object kinds (holes, complex numbers etc.) indicating the sort of objects recognized by that language. Different kinds of objects are each associated with a notion of what it would take from the world for there to count as being an object of that kind: a way of going a possible worlds to a verdict about how many objects of that kind would exist and how those objects behave if that world were actual [5]. Thus, what it takes from the world for there to be a hole will be very different from what it takes for there to be a ball or a company or a wave.

[3]ADD: explanation of quantifier restriction

[4]ADD: explanation of Tarskian rules.

[5]Ultimately we want to be able to give a truth value to any sentence in the language at a given possible world.

If we accept the above idea then we can think of the truth value of a sentence at a given possible world as being determined by a two-step process. First, we use the existence criteria associated with kind terms in the language to populate the domain of a (first order formal) model corresponding to that possible world. Then we apply the usual recursive Tarskian rules to determine the truth conditions for sentences in our language with respect to that model. Languages (understood here to come equipped with an interpretation) can differ from one another with regard to how the first step goes (what model is assigned to a given possible world) while all applying the same Tarskian rules to the resultant models. As a result, there will be a range of senses for the "∃" and "∀" symbols to take on in different languages but all those senses will admit the same inference rules as truth preserving because of the underlying uniformity in the semantics.

4. A Mathematical Model

To make this suggestion more concrete and not subtly incoherent, I will now provide a (simplified) mathematical model for my proposal. The formal structures we introduce below will capture the intended truth conditions of sentences in a language and permit us to give a formal criteria for when a stipulation is acceptable. It is important to note that there is no claim that the natural languages ultimately talk about or gain their meaning from facts about the set theoretic abstract which I will be describing. Indeed it will become clear that we could produce an equally illuminating toy model in any metalanguage that provided a theory (such as category theory, type theory or version of set theory) with sufficient logical strength to represent (in some fashion) the structures used in our formal interpretation[6]. That is

[6]Of course if one was genuinely speaking a language so impoverished that one couldn't represent these structures it might be that one couldn't make sense of statements about the truth of sentences in more ontologically profligate languages but as we only need to

we should merely think of these formal objects as *reflecting* some preexisting structure satisfied by the truth values of sentences in the natural language not creating that structure.

Let L be a first order language without function symbols[7] with certain one place relations distinguished by being classified as kind terms. These kind terms are intended to capture the different magisteria of objects: different kinds of objects which we can choose to start talking in terms of. We can capture the way that a natural language systematically determines truth conditions for sentences by associating L with an interpretation consisting of a sequence of functions $\langle f_{R_1} \ldots f_{R_m} \ldots, g_{c_1} \ldots g_{c_n} \ldots \rangle$ where each of these elements associates some term in the language with a function from possible worlds to abstract mathematical objects or collections thereof, e.g., sets, as follows:

- For each n-place relation symbol R f_R is a function from possible worlds to a set of n-tupples representing the extension of R at that world.
- For each constant symbol c f_c is a *partial* function from possible worlds to abstract objects representing the referent of c at that world.

Furthermore, if we define the domain $D(w)$ associated with a possible world w to be the collection of objects that are members of $f_R(w)$ for some kind term R[8], then at each world w:

make sense of the fact that we seem to be able to cogently talk about such truth this provides no difficulty.

[7]Here I mean language in the special syntactic sense where languages are individuated merely by the symbols they contains and the well formed formulas they allow, and where sentences in languages are only associated with truth conditions under interpretation. Thus L is built up by applying relation symbols to variables and constant symbols and then closing under propositional connectives and both existential and universal quantification.

[8]Intuitively, this insures that every object ranged over by our quantifiers or referred to by some name must belong to at least one kind term in the language.

- For every relation symbol R $f_R(w)$ is a collection of n-tupples of elements from $D(w)$.
- For every constant symbol c $f_c(w)$ is an element of $D(w)$ for each constant symbol c

Considering all these predicates together gives us a function which takes possible worlds to a standard first order mathematical models. The domain of this model at a world w is $D(w)$. The extension of each relation symbol R at w will be $f_R(w)$. Similarly each constant c will be interpreted as $f_c(w)$ at w. Now the truth values for sentences in language L at some possible world w will be given by applying Tarskian rules for determining the truth of a sentence in a model to the model that we have just generated [9]

[ADD example]

4.1. **Stipulation.** Now how do stipulations fit into this picture?

I propose to think of the act of making a stipulative implicit stipulation as an attempt to shift the speaker from one (interpreted) language to another expanded language, in such a way as to ensure that the sentence(s) being stipulated will express a truth, while leaving the meanings of other expressions in the language largely the same.

Now, to answer the bad company objection, we need to explain in a principled and internally consistent fashion what distinguishes acceptable ontologically inflationary stipulations from unacceptable stipulations.

I think at least two restrictions on acceptable stipulations are directly motivated. Firstly, since a stipulative definition tries to specify the truth conditions for claims about a new kind of object in terms of words whose

[9]Note that if a sentence makes reference to a constant symbol c it's truth value will be well defined only at worlds in which f_c is defined. We take this to be a feature of the account as it allows us to represent claims like "The current king of France likes his iPhone." as being neither true nor false in worlds (like our own) lacking a reigning French monarch.[CHECK with lit on non-referring names ...is this the best way of doing things?]

meaning is taken for granted, it should leave the meaning of these other words fixed in a certain sense, e.g., when adding holes to a language and giving truth conditions for claims about holes being located at a given point we don't want to alter facts about whether solid matter is located at a given point.

But what does it mean to hold fixed the meaning of these terms given that one is adding new kinds of objects and specifying the behavior of these terms on these new objects? It would be too strong a condition to require that one cannot change the truth value of *any* sentence which is free of the terms being stipulatively defined. One cannot add objects to the domain associated with a possible world and hold fixed the truth conditions for all sentences built up out of logical connectives and and antecedently meaningful relation symbols. For instance, if we stipulatively add holes to a language whose objects are all either fundamental particles or spatial positions we necessarily change the truth value of the sentence asserting that every object is either a fundamental particle or a spatial location.

However, while we can't ensure that a stipulative change in language won't change the truth values of any sentences in the initial language, we can insist that it produce little or no change to the truth conditions for sentences that occur in ordinary talk or in scientific theories. I will propose a constraint which ensures that an acceptable stipulation can only change the truth conditions of sentences in the initial language with unrestricted quantification[10]. Since most ordinary statements have quantifiers which are restricted to some particular range of kind terms, their truth conditions will not be changed by stipulation[11].

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[10]We say the quantifiers in a formula are all restricted (to kinds) if the formula is either quantifier free or has the form $(\forall x)(P(x) \implies \phi(x))$ or $(\exists x)(P(x) \wedge \phi(x))$ for some kind term P and a formula $\phi(x)$ all of whose quantifiers are restricted to kinds.
[11]CITE

Secondly, the an acceptable stipulation should determine a new, coherent, way of associating possible worlds with domains and choices of extensions for predicates[12]. I propose that there are primitive modal facts about how it would be in principle possible for any objects to be related by any relations, what I will call facts about combinatorial possibility. Similarly, there are modal facts about how it would be combinatorially possible to associate each possible world with some such model[13]. In terms of this notion of combinatorial possibility, we can say the following: an acceptable stipulation must not require that we introduce combinatorially impossible collections of objects like those imagined by naive set theory. It also cannot demand that new objects being stipulatively defined stand in a combinatorially impossible relations to objects countenanced by the old language which we are expanding.

With this in mind I propose the following Harmony condition as a sufficient condition for acceptable stipulations

> **Harmony Condition on Acceptable Stipulations:** A stipulative definition S can be safely added to your language if it would be combinatorially possible to supplement the objects that would already exist at each given possible world with new objects and specify the extension of existing relations on these objects, in such a way as to ensure that S comes out true at all possible worlds *without* changing the extension associated with any kind term in the initial language, changing how any preexisting relation applies to the

[12]Or a range of such ways, in cases where a new stipualtive definition leaves some things vague

[13]I take it that, like facts logical possibility and tautology, the truth of these modal claims does not itself need grounding in the existence of any further objects. See X for much more detail about combinatorial possibility

objects existing prior to the stipulation or violating the constraint that every object must belong to some kind term.

In terms of the formal semantics used above, we can express this idea as follows.

A stipulative definition S adding relation symbols $P_1, \ldots, P_n, \ldots$ and new kind terms $P_{k_1}, \ldots, P_{k_n}, \ldots$, and constant symbols $d_1, \ldots, d_n, \ldots$ to a formal language L with interpretation $I = \langle f_{R_1} \ldots f_{R_m}, g_{c_1} \ldots g_{c_n} \rangle$ is acceptable if there is an interpretation $I' = \langle f'_{R_1} \ldots f'_{R_m} \ldots f'_{P_1} \ldots f'_{P_n} \ldots, g'_{c_1} \ldots g'_{c_n} \ldots g'_{d_1} \ldots g'_{d_n} \ldots \rangle$ of the expanded language L' compatible with S such that for all possible worlds w

- $D_{I'}(w) \supseteq D_I(w)$
- For any k-ary relation symbol R in the initial language $f'_R(w) \cup (D_I(w))^k = f_R$
- For any kind term R in the initial language $f'_{R_{k_i}}(w) = f_{R_{k_i}}(w)$
- For any constant symbol c in the initial language $f'_c = f_c$

Note that by requiring the expanded language have an interpretation the harmony condition trivially disallows stipulations that make incoherent demands about how to extend our existing ways of talking. Furthermore the requirement that relations in the initial language don't change their behavior on the objects given by the initial interpretation ensures that sentences in the initial language involving only restricted quantification don't change their truth value.

The above condition also ensures that the sentences which constitute an implicitly given stipulative definition not only are made true in the resulting expanded interpreted language but also that as long as those sentences only involve restricted quantification they remain true after later stipulative definitions. Hence we can either take all implicitly given stipulations to only apply to existing stipulations (that isif we try to add some concept like set with urelements via the implicitly given stipulative definition that every object belongs to a set we are really only insisting that every object in an existing kind belongs to a set) or allow that some sentences stipulated to be true in an implicit stipulative definition can later be rendered false. However, either choice provides a response to the bad company objection.

5. Eklund's objection

We are now in a position to address Eklund's objection to quantifier variance by providing an account of the sense in which languages more ontologically profligate than own can have true sentences of the form $F(c)$. How can we understand talk of the form " '$F(c)$' is true in L'" where L' admits some new object c not existing in our language, while still representing $F(c)$ as having the logical structure of a predicate applied to an object (as opposed to say merely a string of symbols)?

One might worry that if we don't understand claims about $F(c)$ being true in L' to require that c refer to some existing object (in our current language) then we can't account for our ability as finite beings to learn L'. In other words while it might be possible to give truth conditions for sentences in L' via some arbitrary function mapping the sentences of L' to possible worlds the truth of arbitrary complex sentences couldn't be built up via some finite collection of rules stated in terms of our current language[14] that beings like us can understand.

Furthermore, one might worry that if we don't take sentences in L' to be logically structured in this way we also give up any simple elegant explanation (something other than pure fiat) of why truths in our language

[14]Obviously if we were speaking L' then the intuition that $F(c)$ is true in L' requires that there be some object that c refers to is completely valid. However, the question is how we could learn to speak L' given that we don't yet speak it.

are closed under simple logical inferences. After all if we could move to a language where no such simple closure principles existed then it would be puzzling why our current language had such principles given the vastly larger number of languages that don't satisfy simple closure principles.

The account given above addresses both of these objections by pointing out that we can distinguish the role of the referent of c as the truth maker of $F(c)$ and the work done by the referent of c in giving us a systematic way to understand arbitrarily complex sentences and explain the validity of our rules of logical inference. In the semantics given above we show that we can use an abstract mathematical object to stand in for the referent of c and thereby allowing us to show that meaning in L' is built up systematically in a fashion that accounts for the validity of our rules of logical inference even though that abstract mathematical object isn't the truth maker for $F(c)$ in L'.

6. Limits to stipulation

The harmony condition proposed above also explains why stipulative definition can let us add new mathematical objects like complex numbers and new physical objects like holes but not new animals like Yetis, at least in the sense that would be problematic.

If we try to stipulate the existence of ordinary physical objects like Yetis, stipulations characterizing these objects will not merely require that there be some object in the extension of the Yeti predicate (or even that it have some structure with respect to newly introduced relations) but also that there is an object in the extension of Yeti iff there is an animal with certain properties or some configuration of fundamental particles. Now it wont be combinatorially possible to supplement (at each possible world) the objects we currently acknowledge with Yetis in such a way as to satisfy the claims that there is a Yeti and wherever there is a Yeti there is an very large hairy bipedal primate (or an appropriate configuration of matter or whatever other configuration of objects from preexisting kinds must attain for a Yeti to exist) true while still satisfying the harmony condition since we can't modify the behavior of relations on existing kinds of objects.

One could of course try to stipulate that there are Yetis by simply adding a new kind term, populating it with an object and placing it in the extension of the Yeti predicate while still satisfying the harmony conditions but only at the cost of giving up the idea that there is a Yeti iff there is a very large hairy bipedal primate. In this case our stipulation would move us to a language in which "There is a Yeti." expressed a truth but not in a problematic sense. To see this consider the case where our stipulation introduces a new kind for fictional characters (cf. Amie Tommasson for an example of how one might treat fictional characters as a kind of abstract object?) and places some fictional character in the extension of Yeti. In this new language it would certainly be true that "There is a Yeti." but it would also be true that "Every Yeti is a fictional character." and surely this isn't a troubling case of affairs.

Of course the new kind term might not represent fictional characters but the example illustrates that it isn't a problem merely to move to a language in which "There is a Yeti." expresses a truth. Rather it's only a problem to move to a language where both "There is a Yeti." expresses a truth and our implicit assumptions about the implications of Yeti existence for physical objects (or animals or some other previously populated kind) remain true. However, the harmony condition blocks us from changing facts about already existing kinds so no acceptable stipulation[15] can change the underlying facts denying the existence of Yetis in the problematic sense.

7. Contrast with Plenitudinous Platonism

I will now outline how quantifier variance, spelled out as above, contrasts favorably with a certain natural alternative account called plenitudinous platonism. One might initially hope to reconcile realism about mathematical objects with freedom to stipulate in mathematics without appealing to anything so radical as quantifier variance. Plenitudinous platonism proposes that mathematicians are free to assert the existence of mathematical objects *without* change of language based on considerations of mere coherence because the mathematical universe is so plentiful that mathematical objects exist corresponding to all coherent stipulations[16].

This strategy turns out to face a serious problem: it's not clear whether all the objects that it would intuitively have been acceptable to introduce can consistently exist at the same time. For example, Boolos gives the example of numbers and parities as abstract objects each of which can be coherently posited, but which cannot both exist[17]. The problem is, in essence, that abstraction principles characterizing the intended behavior of the numbers require that there be infinitely many numbers, whereas those for the parities require that every predicate have either an odd or an even number of items falling under it, and hence require that the universe only contain finitely many objects.

This kind of pairwise incompatibility wouldn't pose any problem for a quantifier variance, since quantifier variance allows us to have one language

[15]Assuming the language was already rich enough to represent our reasons for believing Yetis don't exist, e.g., had kind terms that let us describe arrangements of matter.
[16]See CITE Azzuni CITE Eklund
[17]CITE and give quote explaining parities

making the first implicitly defined stipulation true, and a second that makes the second implicitly defined stipulation true, but no third language making both true at once. However, plenitudinous platonism has no such option since all the coherent stipulations must be realized in a single language.

One might hope to impose some stricter constraint than internal consistency on the range of acceptable posits which would secure the needed result that all acceptable mathematical posits are compatible and could hold true in different corners of the same plentiful mathematical universe. However, current attempts to articulate what is required for acceptability as a posit (e.g., by saying that acceptable posits must, in some sense, not impose any restrictions on the size of the universe) have run into trouble over the acceptability of posits characterizing the sets. Furthermore, there are pairs of intuitively acceptable mathematical posits which can be proved *not* to be compatible. For example, Gabriel Uzquiano has recently shown that the axioms of classical meriology and set theory are incompatible, at least in the presence of some natural assumptions[18].

8. Pseudo Questions

Before concluding I will note that in addition to making sense of mathematicians' and sociologists' apparent ability to introduce new kinds of objects quantifier variance promises to let us make sense of certain intuitions to the effect that certain kinds of ontological debates can be defective in a way which would otherwise look quite puzzling.

Take, for example, questions of special composition. Is there really an object which has, as parts, both the Eiffel tower and David Lewis' nose? How exactly do some objects need to be related in order to compose a whole

[18]This result sounds quite surprising but note that Uzquiano takes classical meriology to include the claim that everything can be a part, including abstract objects. He also uses the strong assumption that the sets are to have the same size as the universe as a whole.

which has them as part? Even if you think that other ontological investigations are perfectly reasonable (is there a mind thinking these thoughts which is distinct from the body reading these papers?, are there objects at a temporal remove?) it can be tempting to say that disagreements about these questions of special composition are somehow ill posed. However, it is far from immediately obvious how to cash out this idea. Formally, the question of whether there is a nose-Eiffel tower object seems very similar to uncontroversially meaningful questions like whether there are snails in Antarctica or sentient species on other planets. So what exactly is supposed to be going wrong?

If we accept quantifier variance then we can make sense of the idea that certain ontological debates can be defective on the model of much more familiar ideas about how debates about the exact extensions of certain natural language predicates can be defective. Recall Wittgenstein's example of a chair-like object that flashes in and out of existence every few minutes. Would this item count as a chair? I take it that a standard response to this case is to say that the answer to this question is undefined: all the facts about our minds, our community, natural kinds, the state of the world around us etc. do not suffice to determine an answer. There are there are at least two equally natural ways of precisifying our current notion of chair, C_1 and C_2, such that the relevant object falls in the extension of C_1 but not of C_2. In view of this, it would be a mistake to try to discover by philosophical investigation whether or not the object in question is a chair as that question has no right answer. All we can do to "settle the question" is to insist on one answer or the other, and thereby move to a new language where the string of sounds "is. a chair?" now expresses a different question, one which does have a determinate truth value.

Now quantifier variance claims that there are a range of different but very similar senses available for the quantifiers to express in different languages. If this is right, then we can cash out the idea that some ontological questions are defective or ill-posed in a way that is directly analogous to the familiar story about predicates outlined above. We can say that there are distinct senses of the quantifier E_1 ad E_2 which are both equally natural and fit equally well with our current language use, and that E_1 and $E)2$ assign different truth-values to a target item of ontological dispute like, "There is an object which has both Lewis' nose and the Eiffel Tower as parts". For this reason debate, about whether there is a nose-Eiffel tower object will be nothing but a dispute as to what language we prefer to use. Similarly we can vindicate the weak Carnapain idea that some disputes about the total number of objects an can be ill posed in just the same way (and for essentially the same reasons) as debates about existence can be ill posed.

9. Conclusion

In this paper I proposed an account of how the world makes existential statements in our language true which room for a range of similar but slightly varying senses to be expressed by the quantifiers in different languages. I have argued that the resulting quantifier variance theory allows us to make sense of mathematicians' apparent ability to stipulate, while avoiding known problems about about bad company and truth conditions for statements in more ontologically profligate languages.