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RESEARCH STATEMENT

My work focuses on epistemology and the philosophy of mathematics, with special emphasis on how understanding mathematical knowledge can illuminate broader philosophical issues in philosophy of language, metaphysics and metaethics.

Much of my research grows out of (but does not depend on) a longtime project to answer classic **access worries about mathematical knowledge -** in a way that combines appealing features of traditionally opposed, rationalist, empiricist, and conventionalist approaches.

1. The Mathematical Access Problem

A common objection to (realist) philosophical claims about morals, aesthetics and metaphysics is that even if there were facts of that kind we couldn't possibly know what they are. Understanding when such concerns, broadly deemed access worries, justify rejecting any claim to (non-trivial) knowledge about some domain is essential to adequately answer the 'big' questions – What, if anything, is morally right and wrong? Is some art 'objectively' better or is it all just a matter of taste? Are there truths about nature which go beyond patterns in our experience?

Access worries arise when realist views about a domain make it hard to explain how humans could reliably form accurate beliefs about that domain—except by assuming a suspiciously lucky coincidence, e.g., "Since moral facts (as the realist understands them) don't have any causal influence, even if they existed how could we possibly know what they were?" This makes the case of mathematics particularly interesting, since neither mathematical objects or facts have any causal influence on us yet even most philosophers who are happy to deny there are (realist) moral or aesthetic facts are reluctant to deny there are mathematical facts.

Before briefly describing my work on this problem and various related ones below, it's worth clarifying what I mean by the access problem for (realist) mathematical knowledge. While my view is not too far from standard Platonism, I'm not defending strong Platonic claims that numbers and sets exist 'in exactly the same sense' as tables and chairs (though I do believe mathematical objects can be said to literally exist). Nor am I merely defending the view that if we stipulate some formal axiom system there will be facts about what can be proved in that system. Rather, I am defending a view which is truth-value realist in the sense that it holds there are definite facts about many mathematical claims which are neither provable nor disprovable.

1.1. **Posing The Access Problem.** In "Coincidence Avoidance and Formulating the Access Problem" I defend traditional access worries about mathematical knowledge against some recent challenges, and try to clarify what it would take to solve them. If access worries were a mere skeptical demand to show that your mathematical/moral beliefs followed from indubitable premises they would not be very troubling. For, as Descartes showed us, it's in principle possible to doubt almost anything.

Instead, following a suggestion by Field 1980, I think access worries reflect an internal tension within the realist's own theory. In "Coincidence Avoidance and Formulating the Access Problem" I argue that this internal tension is best understood in terms of *informal coincidence-avoidance intuitions* (that are widely accepted by the realist as guiding theory choice in other domains[1]). The access worrier can be understood as arguing that *on the realist's own theory* it would be an unbelievable coincidence if our beliefs about the domain (e.g. our mathematical beliefs) were correct.

Accordingly, one can address access worries by giving a toy model: a simplified, illustrative explanation that shows how creatures like us might come to know truths in the domain without invoking any unexplained miracle.

Such toy models are common in other domains. We might, for example, persuade someone that a wind-powered vehicle can go faster than the wind by using an idealized model that ignores friction and treats air as a stream of point particles. Likewise, in the case of mathematical knowledge, a toy model that illustrates a plausible kind of explanation for our accuracy—while abstracting away from real-world messiness—can go a long way toward dissolving the sense that our knowledge is a spooky coincidence.

To answer access worries the realist needn't eliminate all mystery, but only must show that belief–truth alignment in the domain is not in principle inexplicable. Even a schematic or idealized story can suffice, so long as it defuses the tension.

1.2. **Solving The Access Problem.** In papers like "Mathematical Access Worries and Accounting for Knowledge of Logical Coherence" and "(Probably) Not Companions In Guilt" I propose a solution to the access problem for mathematical

[1] I think this (seemingly trivial) point has important consequences as it blocks the common tactic of using the difficulty of conceptually analyzing the notion of coincidence to resist access worries.

realists which combines elements from traditionally opposed rationalist, empiricist and conventionalist approaches to mathematical knowledge.

One intuition motivating my proposed solution to the mathematical access problem is the conventionalist insight that a great deal of mathematical knowledge is true as a matter of convention and/or stipulation. There is no great mystery to how mathematicians know that “Even numbers are divisible by 2,” that’s just what mathematicians choose to mean by the term ‘even’. If mathematicians had instead defined ‘odd’ to mean ‘divisible by 2,’ then mathematicians would have believed—and it would have been true—that “odd numbers are divisible by 2” (since in that scenario, the sentence would have expressed a different, true proposition).

However, this kind of appeal to the possibility of knowledge by convention and language change can’t explain all of our mathematical knowledge (see the discussion of truth-value realism in §3). For example, we use mathematical claims in the sciences in ways that would not be safe if mathematicians made incoherent posits that implied contradiction (and therefore let one derive everything).

However, I argue, we can reduce the access problem for mathematical knowledge to an access problem for knowledge of logical coherence, (i.e., it’s enough to explain how mathematicians seem to know that certain axiom systems are coherent). I suggest formalizing the latter knowledge in terms of knowledge of the logical possibility (possibility with respect to the most general, subject-matter neutral constraints on how any objects could be related by any relations) of a structure which satisfies the axioms. Thus, by combining conventionalism with rationalist appeal to a faculty of good a priori reasoning about logical possibility, we can give a kind of partial answer to mathematical access worries. However, this leaves us with a residual access problem —accounting for how our accuracy in reasoning about logical possibility need not require a spooky coincidence.

To answer this residual access problem, I take inspiration from another major strand of thought in the philosophy of mathematics – empiricism. Quine’s classic empiricist proposal that we gain knowledge of mathematical objects in the same way as we learn about other objects of our best scientific theory faced worries about how to make sense of ‘recreational’ and ‘higher’ mathematical knowledge that isn’t necessary to formulate our best scientific theory. But I suggest we can avoid these problems by taking correction by experience to lead us to correct general methods of reasoning about logical possibility (which may recognize the logical possibility of pure mathematical structures quite unnecessary to the sciences).

I present a toy model illustrating how creatures like us might acquire reliable methods for identifying logical possibility through mechanisms like abduction and cultural selection. I argue that our ability to gain true beliefs about logical possibility from knowledge of concrete actual situations is no more mysterious than our ability to gain knowledge of what's physically or chemically possible. For knowledge of actual facts about non-mathematical objects can give us initial data points about what's possible, from which to abductively generalize correct laws and good inference methods.

Though I trace the origins of our logical reasoning methods to naturalistic processes (e.g., evolution, cultural selection), I do not endorse mathematical empiricism. Rather, I maintain that once acquired, these methods yield a priori knowledge in the familiar foundationalist sense: justified without further empirical input. I defend a modestly deflationary view of a priori justification (discussed in §4), on which we have default warrant to make any logically valid inference we find immediately compelling.

In sum, my approach combines: (1) conventionalist insights about how mathematical definitions create knowledge by stipulation, (2) rationalist recognition of our ability to reason about logical possibility, and (3) empiricist explanations of how experience helps us develop reliable methods for such reasoning. This synthesis removes the appearance of miraculous alignment between mathematical truth and human belief without sacrificing mathematical realism.

I'm also interested in answering access worries in other domains, in ways that (somewhat) parallel my approach to mathematical access worries. For example, in "(Probably) Not Companions In Guilt" I argue that (contra popular "companions in innocence" defenses of moral realism) traditional moral realists can't solve their access problem using a version of the strategy I propose for answering mathematical access worries. And in "Metaethical Deflationism, Access Worries and Motivationally Grasped Oughts", I argue that such an approach is open to metaethical deflationists – and sketch a theory of moral notions as motivationally grasped concepts to facilitate giving such an account.

1.3. **Ongoing work.**

- A paper developing a unified account of metaphysical possibility, combining a functional role analysis, an analysis of metaphysical possibility talk in terms of facts about logical possibility and facts about the actual world[2], and a theory of our epistemic access to such facts.

[2]My paper "Gunk mountains: A Puzzle" arose from noticing a road block along the way to this project.

- A book project unifying and more fully developing all the different elements of my proposed answer to mathematical access worries.

2. Foundations of Set Theory and A Puzzle About Height

Set theory is widely regarded as the foundational language of mathematics. It allows us to interpret virtually all mathematical talk—about numbers, functions, or spaces —as talk about the cumulative hierarchy of sets. This hierarchy grows by repeatedly forming collections of earlier sets, layer by layer. There is a fairly widely agreed on story about how this structure is supposed to be built up at each level, but a persistent puzzle arises when we ask: how far does this process go?

In other words, what is the intended height of the hierarchy of sets? Naively, one might say it goes "all the way up." But taken at face value, such views lead to paradox. And unlike with the natural numbers or real numbers, there's no clear, uniquely characterized structure that everyone agrees set theory is about. Multiple set-theoretic universes—each satisfying the standard axioms (ZFC)—are possible, and it's unclear which, if any, is the "correct" one.

This presents a deep philosophical challenge for those who think such mathematical claims have objective truth values. If we can't pin down a single universe of sets, what are set-theoretic statements really about?

My research develops and defends a potentialist alternative. On this view, set theory isn't about a completed infinite universe of sets, but about how finite or partial universes could be extended. Rather than trying to identify the true universe of sets, we explore which kinds of extensions are possible, according to the structural rules governing set formation.

In the first half of my book, *A Logical Foundation for Potentialist Set Theory*, I develop a novel logical framework for this idea[3]. I propose a modal logic for a special generalization of the notion of logical possibility mentioned above, and use this to formulate potentialist translations of set theoretic sentences.

I provide a formal interpretation of standard set theory in my system – thereby showing it is sufficient to capture all normal mathematical reasoning. In addition to justifying the axioms of set theory, (I argue) this framework helps clarify the explanatory role mathematics plays in the sciences and supports traditional expectations that mathematics and logic are closely related—and that mathematics is, in a deep sense, the science of structure.

[3] I argue that this version of potentialism has various advantages over prior formulations of potentialist set theory. For example, it avoids 'quantifying in' (making claims about what is logically possible for specific objects), and avoids appeal to a sui generis 'interpretational possibility' operator which previous developments by Linnebo and Studd have had to employ.

In response to getting some technical questions about the appendices in the book (recently taught in seminars by Chris Scambler at Oxford and Andrew Bacon at University of Southern California) and a planned volume on my book organized by Nikolaj Pedersen for the Asian Journal of Philosophy, I've developed a formal Lean proof checker for the system in my book, in hopes of making the formal proposals in the book more clear and engaging, and as a first step towards AI assisted translation of the natural language arguments in these appendixes.

2.1. **Ongoing work.**

- A paper (R&R at Philosophia Mathematica) contrasting my favored form of potentialism with certain popular alternatives.
- A consistency strength analysis (formalized in Lean) showing how my logic of logical possibility aligns with standard set-theoretic frameworks.
- A paper raising some philosophical questions and challenges for attempts to use reflection principles in set theory to justify large cardinal axioms.

3. Mathematical Truth Value Realism

If the above answer to access worries is right, it succeeds in removing or significantly reducing a certain kind of internal tension facing mathematical truth-value realist. But is there any positive reason to be a mathematical truth-value realist? That is, why think there are definitive answers to mathematical questions like the continuum hypothesis, which cannot be resolved by proofs from the standard ZFC axioms for set theory?

Several of my papers argue that rejecting truth-value realism forces radical—and arguably implausible—revisions to our understanding of the relationship between mathematics, logic, and certain kinds of (seemingly) necessary constraints on the behavior of non-mathematical objects.

For example, in "Malament-Hogarth Machines and Tait's Axiomatic Conception of Mathematics", I defend truth-value realism by arguing (contra suggestions that ordinary mathematical practice doesn't suggest mathematical truth to outrun proof) that ordinary mathematical practice includes willingness to accept some mathematical statements whose answers cannot be determined by proofs we could discover, in response to hypothetical experiments involving computers and black holes. In "Physical Possibility and Determinate Number Theory", I argue that our capacity to refer to a favored notion of physical possibility may be enough to resist skepticism about whether we're thinking about the intended model of arithmetic. And in "${\Sigma^0}_1$ Soundness isn't Enough: Number-theoretic Indeterminacy's Unsavory

Physical Commitments" I argue that (under certain plausible assumptions) taking us to be free stipulate answers to formally undecidable questions about numbers commits one to dogmatically ruling out certain intuitively possible configurations of concrete objects. In "Explanatory Indispensability and the Set Theoretic Multiverse", I suggest that forms of truth-value anti-realism that reject the idea that set theory encapsulates all possible ways of choosing (such as Hamkins' multiverse theory, which accepts the existence of sets but rejects mathematical truth-value realism by denying there's a unique intended hierarchy of sets in favor of a multiverse of equally intended hierarchies which differ on the truth-value of questions like CH) prevent us from making sense of certain seemingly cogent possible explanations for physical facts.

3.1. **Ongoing work.**

- A positive account of our ability to refer with mathematical concepts in the ways expected by truth-value realism, supplementing my earlier criticisms of anti-realist views. I advocate a version of McGee's appeal to open-ended acceptance of axiom schemas McGee 1997 revised to address some objections I've raised to McGee's formulation. I contrast these ways of using expectations about physical facts to support reference and truth-value realism with Warren's use of similar thought experiments to argue that we accept infinitary inferences.

4. Basic a Priori Knowledge and Epistemic Unprincipledness

Consider two valid deductive inferences: (1) deriving "B" from "A" and "If A then B" versus (2) deriving Fermat's Last Theorem from the ZFC axioms—a multi-step process requiring hundreds of pages of advanced mathematics. Both inferences are equally valid and truth-preserving, yet we treat the first as immediately justified, while viewing the second as requiring substantial work to justify. What accounts for this difference?

In papers like "External World Skepticism, Confidence and Psychologism about the Problem of Priors" and "Default Reasonableness and the Mathoids", I argue for a surprising conclusion: there is nothing intrinsically special that distinguishes the collection of logically valid inferences we treat as epistemically basic – or the class of propositions that should be assigned high probability a priori. Instead, our standards for acceptable inference and correct priors partly reflect contingent facts about human psychology (e.g., facts about which patterns of reasoning we find immediately compelling or cognitively effortless) – just as perhaps the boundaries of the concept 'edible' reflect contingent features of what humans can digest rather than tracking some deeper biological natural kind. Taking this view allows us to make sense of a wide range of intuitive epistemic judgments, without requiring a principled, psychology-independent solution to problems like the problem of priors (which probabilities are rational to hold in advance of all evidence and why?).

4.1. **Ongoing work.**

- A paper further developing my epistemic unprincipledness view as a third path between epistemic consequentialism and deontology, addressing worries that this view somehow can't account for the deliberative roles for our thoughts about moral and epistemic oughts.

5. Ontological Knowledge by Convention, More Generally

A final line of my research concerns the possibility of language change which gets us talking in terms of new kinds of objects. It seems like sometimes we can introduce new objects merely by linguistic stipulation, e.g., we can say that a road is to count as containing a pothole if and only if it is indented sufficiently steeply, and thereby come to start talking in terms of new objects (potholes).

But this idea raises some puzzles. For example, how can we state this idea (or some version of it able to do philosophical work like solving access worries) without paradoxically claiming that there are objects we're not now quantifying over but could begin talking about? And if we allow the possibility of gaining easy knowledge of objects to explain knowledge of which logically coherent pure mathematical axioms are true, or how indented a road has to be to contain a pothole, wouldn't this have implausible implications like letting us account for purported knowledge of fairies in a similar way?

In work like "Chalmers, Quantifier Variance and Mathematicians' Freedom" and chapter 16 of *A Logical Foundation for Potentialist Set Theory*, I argue that we can use the generalized logical possibility operator referenced above to non-paradoxically describe starting to talk in terms of more objects, and I sketch an epistemic dynamics for stipulation (how acts of stipulative redefinition that change our language can change our knowledge and other epistemically valuable states, like having short terms that express joint-carving, abduction-friendly properties and having reliable observation procedures for applying the terms in our language). Using this, I advocate a rational reconstruction framework for generally evaluating how much work appeals to the possibility of language change and knowledge by convention can do to help answer access worries.

5.1. **Future directions.**

- A metaontology paper further developing the above view, with special focus on answering the "bad company" challenge referenced above (why couldn't we say the same about fairies?), and arguing that my view avoids certain problems (awkward commitment to a bright line distinction between 'trivial' and 'substantive' facts) for more radical approaches to ontological knowledge by convention, which aim to debunk metaphysics.
- (Tentative) A conceptual engineering paper drawing on insights from programming language design (e.g., object inheritance) to model how social conventions stabilize reference to newly introduced abstracta.

Outside of these core research areas, I have diverse interests. For example, I have draft papers on neoplatonic philosophy of love, Disraelian social epistemology, the philosophy of probability and certain practical issues connecting AI development with epistemology.

References

Berry, Sharon (2013). "Default Reasonableness and the Mathoids". In: *Synthese* 190.17, pp. 3695–3713.

– (2014). "Malament-Hogarth Machines and Tait's Axiomatic Conception of Mathematics". In: *Erkenntnis* 79.4, pp. 893–907.

– (2015). "Chalmers, Quantifier Variance and Mathematicians' Freedom". In: *Quantifiers, Quantifiers, and Quantifiers: Themes in Logic, Metaphysics, and Language*. Springer, pp. 191–219.

– (2018). "(Probably) Not Companions In Guilt". In: *Philosophical Studies* 175.9, pp. 2285–2308.

– (2019a). "External World Skepticism, Confidence and Psychologism about the Problem of Priors". In: *The Southern Journal of Philosophy* 57.3.

– (2019b). "Gunk mountains: A Puzzle". In: *Analysis* 79.1, pp. 3–10.

– (2020). "Coincidence Avoidance and Formulating the Access Problem". In: *Canadian Journal of Philosophy*, pp. 1–15.

– (July 2021). "Physical Possibility and Determinate Number Theory". In: *Philosophia Mathematica* 29.3, pp. 299–317. DOI: `10.1093/philmat/nkab013`.

– (2022). *A Logical Foundation for Potentialist Set Theory*. Cambridge: Cambridge University Press. ISBN: 978-1-108-83431-5. DOI: `10.1017/9781108992756`.

– (June 2023). "${\Sigma^0}_1$ Soundness isn't Enough: Number-theoretic Indeterminacy's Unsavory Physical Commitments". In: *British Journal of Philosophy of Science* 74.2, pp. 469–484. DOI: `10.1086/714813`.

Berry, Sharon (July 2024). "Metaethical Deflationism, Access Worries and Motivationally Grasped Oughts". In: *Ethical Theory and Moral Practice* 27.3, pp. 301–318. ISSN: 1572-8447. DOI: 10.1007/s10677-023-10423-y.

– (n.d.[a]). "Explanatory Indispensability and the Set Theoretic Multiverse". In: *Synthese*. forthcoming.

– (n.d.[b]). "Mathematical Access Worries and Accounting for Knowledge of Logical Coherence". In: *The Journal of Philosophy*. forthcoming.

Field, Hartry (1980). *Science Without Numbers: A Defense of Nominalism*. Princeton University Press.

McGee, Vann (1997). "How We Learn Mathematical Language". In: *Philosophical Review* 106.1, pp. 35–68.